

# ميكانيك الموائع الحيوية

## Biofluid Mechanics



جامعة ساوة الأهلية

الكلية التقنية الهندسية

قسم هندسة تقنيات الفيزياء الطبية والعلاج الاشعاعي

المرحلة الثانية

السنة الدراسية: 2025 – 2026 / الفصل الأول

المحاضرة الثانية: اجهاد القص واللزوجة - 1

## Concept of Stress

Stress, like pressure, is a term used to describe the *intensity of a force*—the quantity of force that acts on a unit of area. Force, in structural design, has little significance until something is known about the resisting material, cross-sectional properties, and size of the element resisting the force (Figure 5.7).

The unit stress, or the average value of the axial stress, may be represented mathematically as

$$f = \sigma = \frac{P}{A} = \frac{\text{axial force}}{\text{perpendicular resisting area}}$$

where

$P$  = applied force or load (axial); units are expressed as #, kips (k), N, or kN

$A$  = resisting cross-sectional area perpendicular to the load direction; units are expressed as in.<sup>2</sup>, ft.<sup>2</sup>, m<sup>2</sup>, or mm<sup>2</sup>

$f = \sigma$  (sigma) = the symbol(s) representing unit stress (normal); units are expressed as #/in.<sup>2</sup>, k/in.<sup>2</sup>, k/ft.<sup>2</sup>, and pascal (N/m<sup>2</sup>) or N/mm<sup>2</sup>

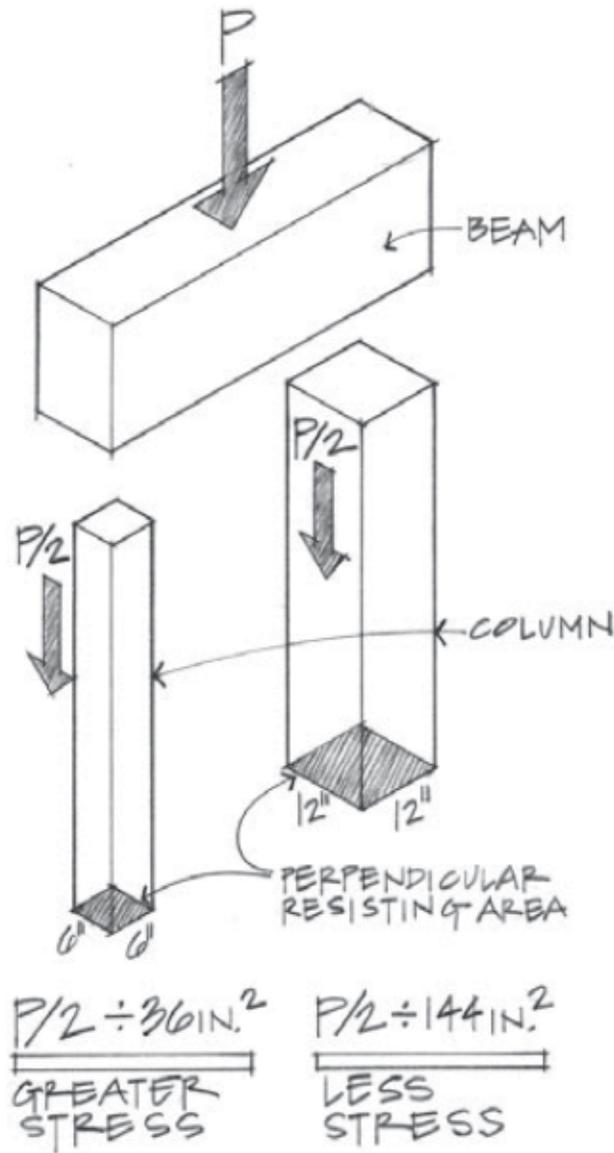


Figure 5.7 Two columns with the same load, different stress.

- Axial stress can either be **Compressive** or **Tensile**.

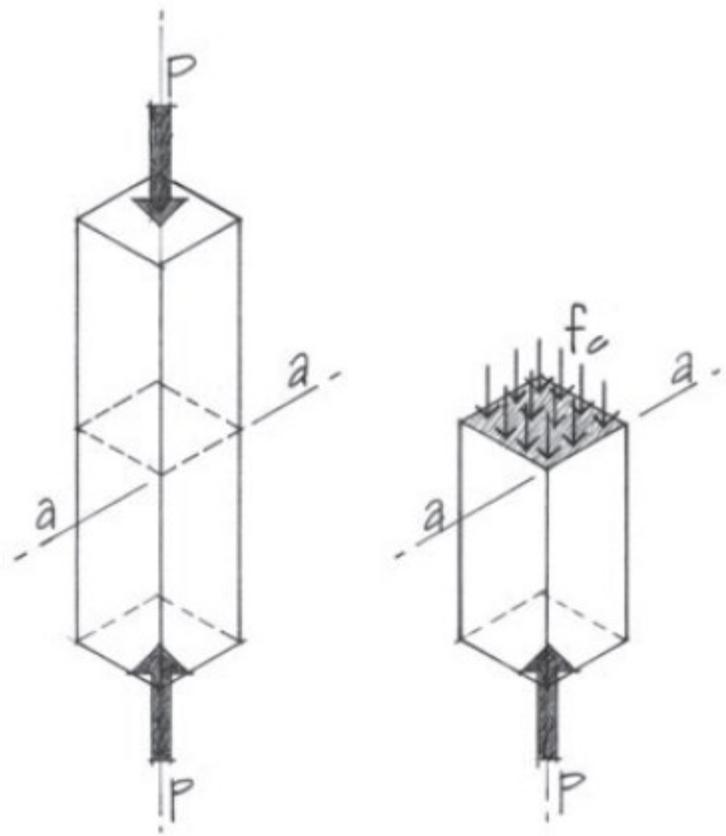


Figure 5.8 Normal compressive stress across section a-a.

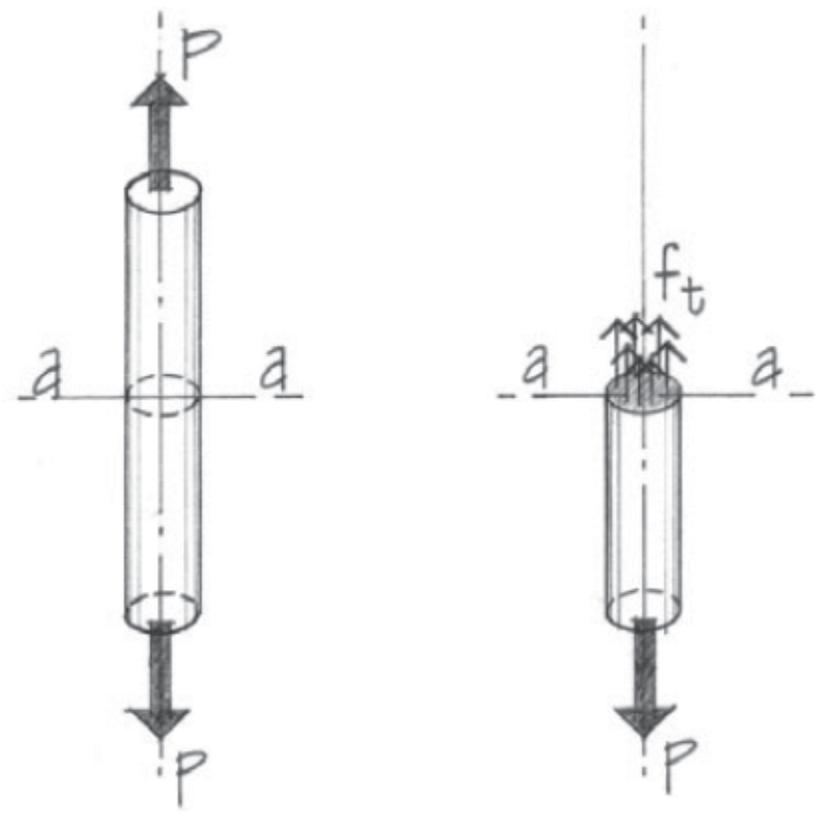


Figure 5.9 Normal tensile stress through section a-a.

**Example:** Two circular columns, each support a compressive force of 1500 kN.

- Column 1 has a diameter of 25 cm.
- Column 2 has a diameter of 65 cm.

Find the stress on each column.

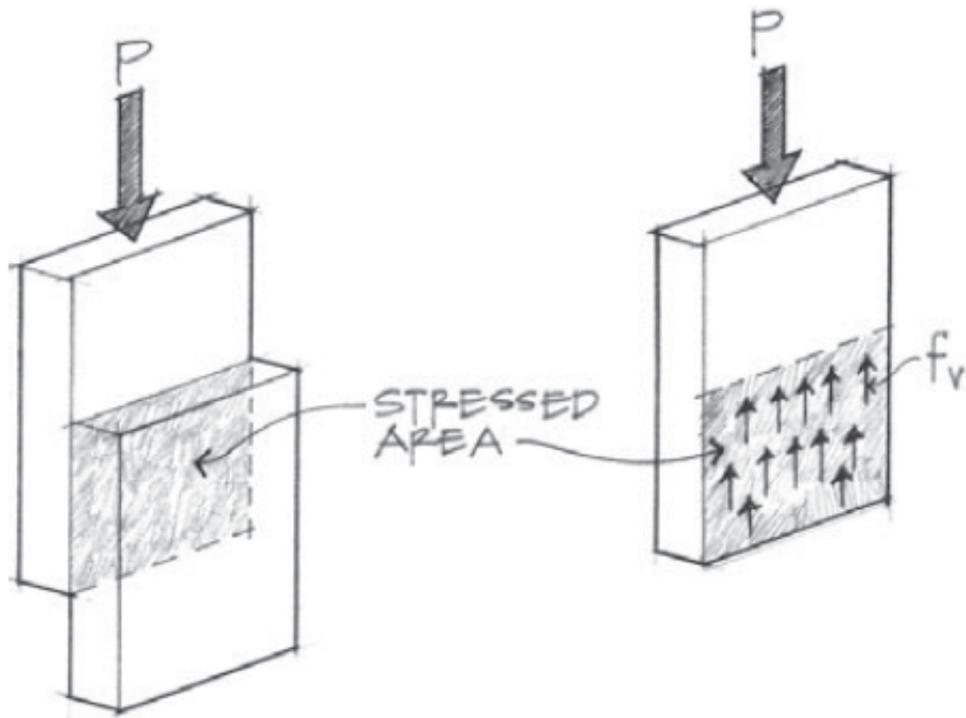


Figure 5.10 Shear stress between two glued blocks.

## Shear stress

*Shear stress*, the second classification of stress, is caused by a tangential force in which the stressed area is a plane parallel to the direction of the applied load. (Figures 5.10 through 5.12). Average shear stress may be represented mathematically as

$$f_v = \tau = \frac{P}{A} = \frac{\text{axial force}}{\text{parallel resisting area}}$$

where

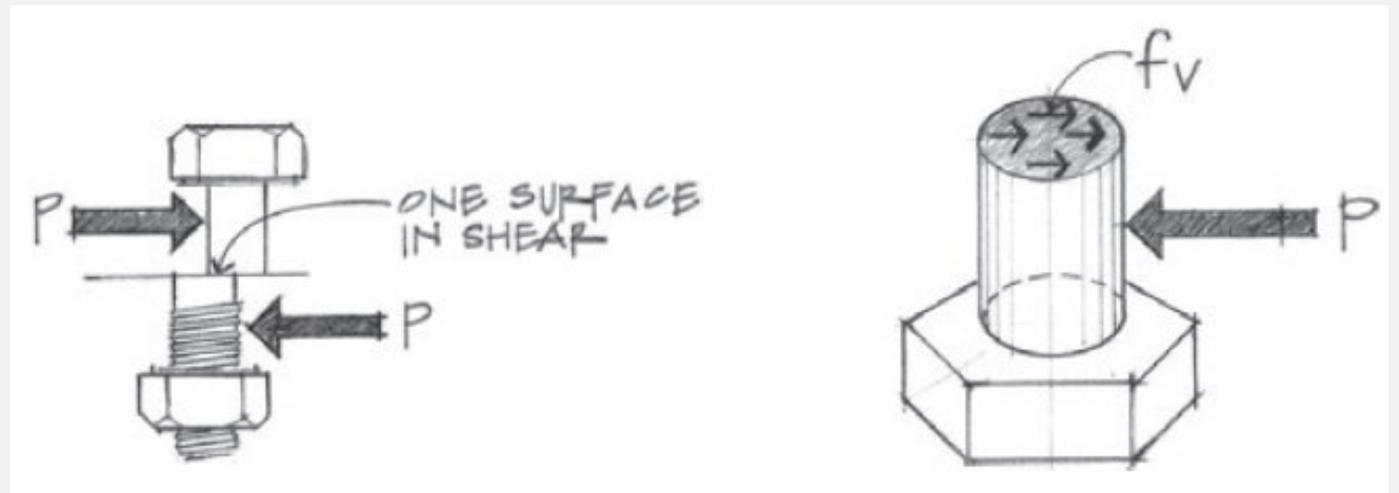
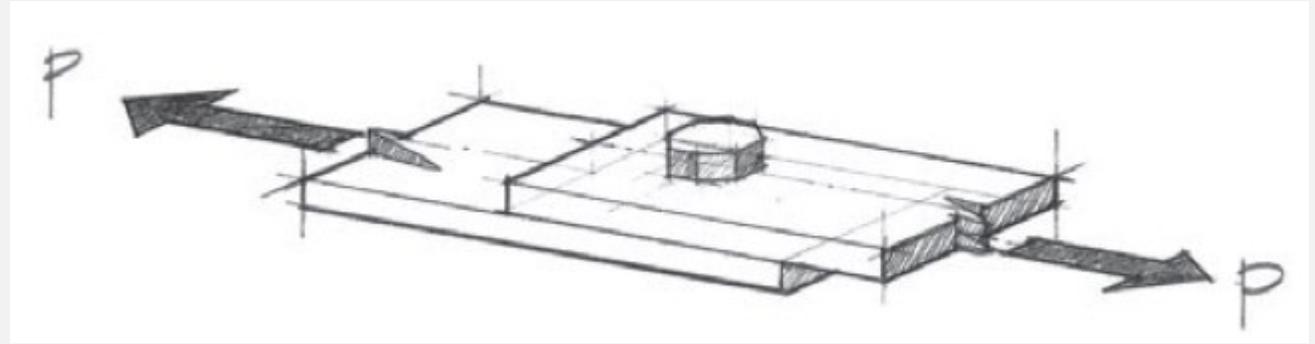
$P$  = applied load (# or k, N or kN)

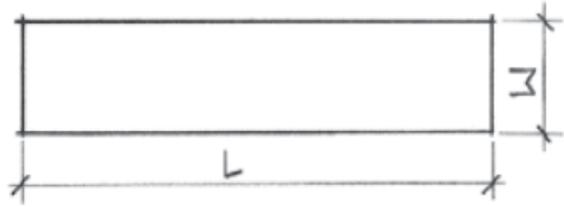
$A$  = cross-sectional area parallel to load direction  
(in.<sup>2</sup>, m<sup>2</sup>, mm<sup>2</sup>)

$f_v$  or  $\tau$  = average unit shear stress (psi or ksi, N/mm<sup>2</sup>)

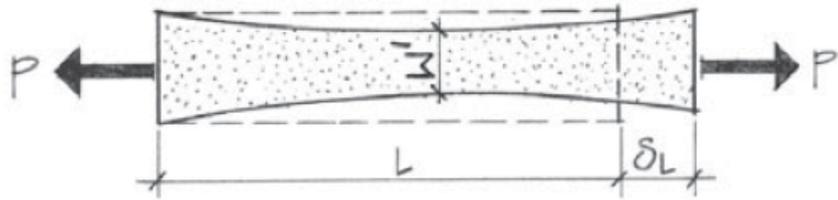
## Example of a shearing stress problem

Find the shear stress on the bolt if the force  $P$  is 750 N, and the bolt diameter is  $\frac{1}{2}$  inches.





(a) Sheet of rubber—unloaded.



(b) Sheet of rubber—under load.

Figure 5.15 Deformation of a sheet of rubber.

## Deformation and Strain

Most materials of construction deform under the action of loads. When the size or shape of a body is altered, the change in any direction is termed *deformation* and given the symbol  $\delta$  (delta). *Strain*, which is given the symbol  $\epsilon$  (epsilon) or  $\gamma$  (gamma), is defined as the deformation per unit length. The deformation or strain may be the result of a change of temperature or stress.

Consider a piece of rubber being stretched:

$L$  = Original length

$W$  = Original width

$W'$  = New width

$\delta_L$  = Longitudinal change in length (deformation)

$W - W' = \delta_t$  = Transverse change in length

Strain resulting from a change in stress is defined mathematically as

$$\epsilon = \frac{\delta}{L}$$

where

$\epsilon$  = unit strain (in./in.)

$\delta$  = total deformation (in.)

$L$  = original length (in.)

# Shear Strain

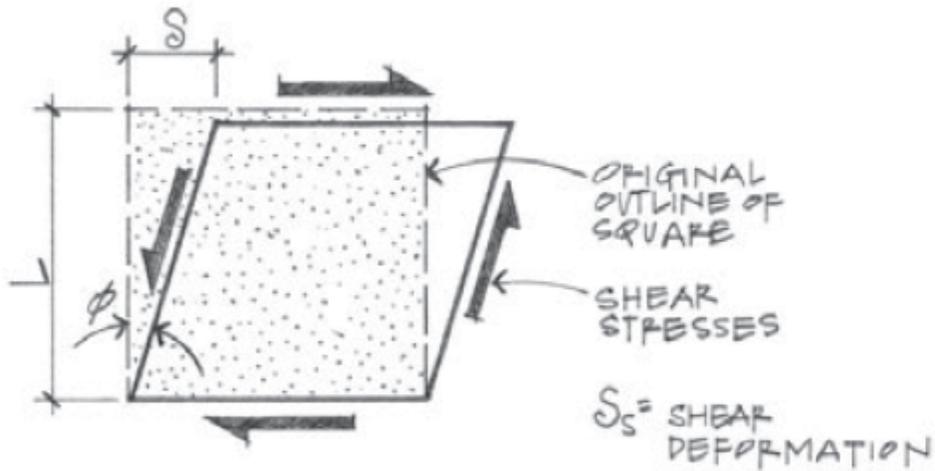


Figure 5.16 Shear deformation.

Members subjected to a shear stress undergo a deformation that results in a change in shape.

Rather than an elongation or shortening, shearing stress causes an angular deformation of the body. The square shown in Figure 5.16 becomes a parallelogram when acted upon by shear stresses. Shearing strain, represented by  $\gamma$ , is

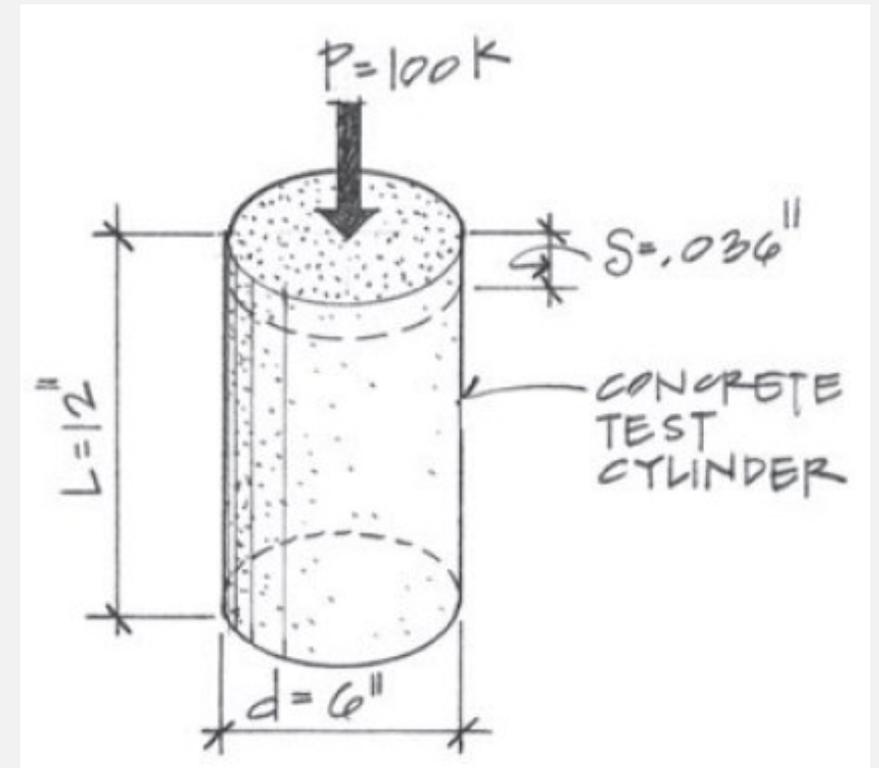
$$\gamma = \frac{\delta_s}{L} = \tan \phi \cong \phi$$

When the angle  $\phi$  is small,  $\tan \phi \cong \phi$ , where  $\phi$  is the angle expressed in radians.

**Example:** A cylinder 6" in diameter and 12" in height is loaded with  $P = 100$  k, and the resulted shortening is 0.036 inch. Determine the strain in the cylinder.

Notes:

- 1.0 k (kilopound) = 1000 lb
- 1.0 lb = 4.448 N



**Example:** A thin rectangular plate, with dimensions 200 mm by 150 mm, is deformed by a shear force. After deformation, the top edge of the plate is displaced by 0.25 mm horizontally relative to the bottom edge. Determine the shear strain.

# Understanding Viscosity: Displacement and velocity

To understand viscosity, let us begin by imagining a hypothetical fluid between two parallel plates which are infinite in width and length. See Fig. 1.4.

The bottom plate A is a fixed plate. The upper plate B is a moveable plate, suspended on the fluid, above plate A, between the two plates. The vertical distance between the two plates is represented by  $h$ . A constant force  $F$  is applied to the moveable plate B causing it to move along at a constant velocity  $V_B$  with respect to the fixed plate.

After some infinitesimal time  $dt$ , a line of fluid that was vertical at time  $t = 0$  will move to a new position, as shown by the dashed line in Fig. 1.4. The angle between the line of fluid at  $t = 0$  and  $t = t + dt$  is defined as the shearing strain. Shearing strain is represented by the Greek character  $\gamma$  (gamma).

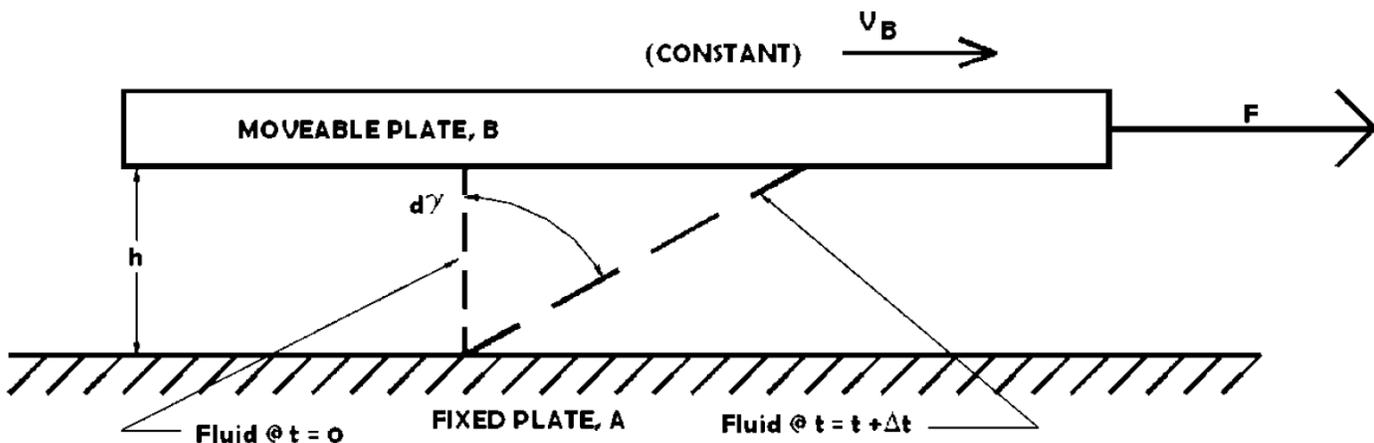


Figure 1.4 Moveable plate suspended over a layer of fluid.

The first derivative of the shearing strain with respect to time is known as the **rate of shearing strain  $d\gamma/dt$** .

$$\tan(d\gamma) = \frac{\text{opposite}}{\text{adjacent}} = \frac{V_B dt}{h}$$

For small displacements,  $\tan(d\gamma)$  is approximately equal to  $d\gamma$ .

$$d\gamma/dt = V_B/h$$

The rate of shearing strain is also denoted by  $\dot{\gamma}$ , and has the units of 1/s.

## مقدمة الى مفهوم اللزوجة

لكي نفهم معنى اللزوجة، نتخيل سائلاً محصوراً بين صفيحتين أفقيتين متوازيتين لا نهائيّتين في الطول والعرض.

• الصفيحة السفلية  $A$  ثابتة.

• الصفيحة العلوية  $B$  متحركة بسرعة ثابتة  $V_B$  نتيجة تطبيق قوة  $F$ .

• المسافة بين الصفيحتين تمثل بـ  $h$ .

عندما تتحرك الصفيحة العلوية، تتحرك معها الطبقات العليا من السائل بينما تبقى الطبقات القريبة من الصفيحة الثابتة ساكنة تقريباً. بعد زمن صغير جداً  $dt$ ، تميل خطوط الجزيئات السائلة لتكوّن زاوية صغيرة  $d\gamma$  مع الوضع العمودي الأصلي، وتسمى هذه الزاوية إنفعال القص - **Shearing Strain**.

### التفسير الفيزيائي

يوضح هذا التحليل أن سرعة طبقات السائل تتدرج خطياً من الصفر عند الصفيحة الثابتة إلى السرعة  $V_B$  عند الصفيحة المتحركة. ويمثل هذا التدرج في السرعة معدل تغير إنفعال القص، وهو مقياس أساسي لتحديد اللزوجة، أي مقاومة السائل لتغير شكله تحت تأثير إجهاد القص.

### مفاهيم أساسية

المصطلح بالإنجليزية	المعنى بالعربية	الرمز	ملاحظات
Shearing Strain	إنفعال القص	$\gamma$	الزاوية الناتجة عن إزاحة طبقات السائل
Rate of Shearing Strain	معدل تغير إنفعال القص	$\dot{\gamma} = \frac{d\gamma}{dt}$	التغير في إنفعال القص مع الزمن
Plate Velocity	سرعة الصفيحة المتحركة	$V_B$	ناتجة عن القوة المطبقة $F$
Distance between Plates	المسافة بين الصفيحتين	$h$	تؤثر عكسياً على معدل القص

### \* لاحظ

لو كانت المادة بين الصفيحتين

صلبة فإن سلوكها اثناء تعرضها

لاجهاد القص سيكون مختلفاً،

حيث ستعاني المادة من التشوه

ولمسافة معينة محدودة ثم تتوقف

الشريحة العلوية عن الحركة.

أما في حالة المادة السائلة فستكون

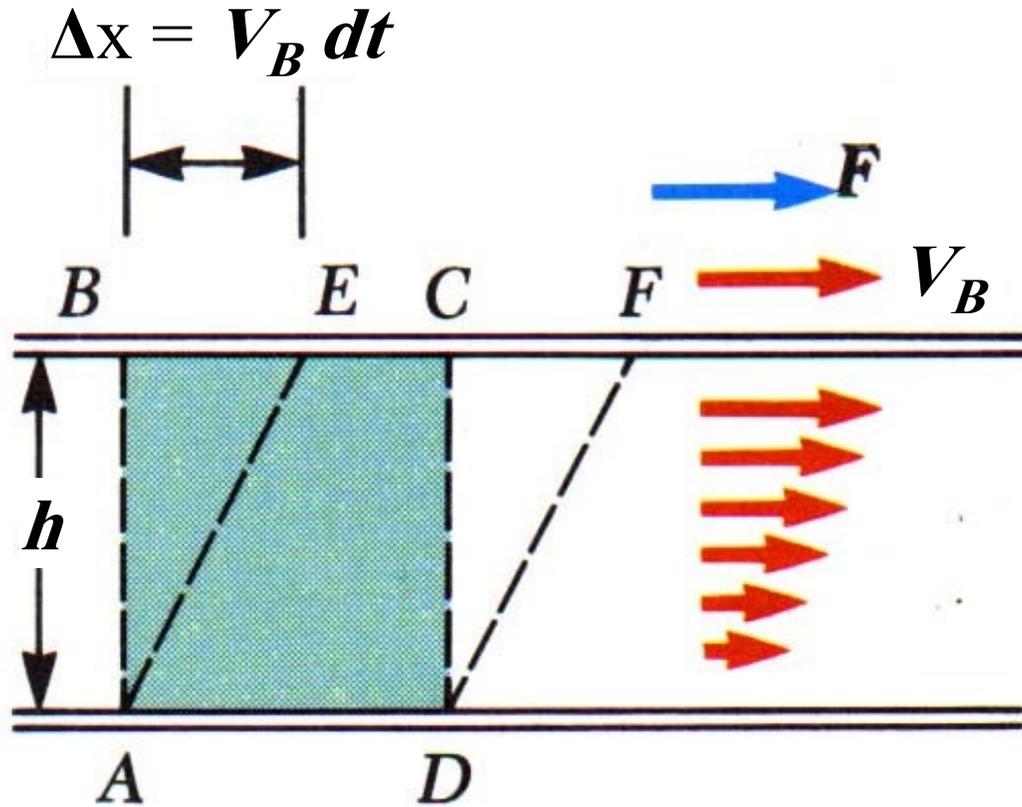
حركة الصفيحة العلوية مستمرة،

لأن تعريف المادة السائلة هو أنها

تشوه باستمرار تحت تأثير إجهاد

القص (مهما صغرت قيمة هذا

الاجهاد).



$$\tan(d\gamma) = \frac{V_B dt}{h} \quad (1)$$

وبما أن الزاوية صغيرة، يمكن تقريب:

$$d\gamma \approx \frac{V_B dt}{h} \quad (2)$$

ومنها يمكن اشتقاق المعدل الزمني لتغير إنفعال القص - Rate of Shearing Strain

$$\frac{d\gamma}{dt} = \frac{V_B}{h} \quad (3)$$

ويرمز له أيضًا بالرمز:

$$\dot{\gamma} = \frac{V_B}{h} \quad (4)$$

ووحدة قياسه هي:

$$[\dot{\gamma}] = s^{-1}$$

Let the distance from the fixed plate to some arbitrary point above the plate be  $y$ . The velocity  $V$  of the fluid between the plates is a function of the distance above the fixed plate A. To emphasize this we write

$$V = V(y)$$

The velocity of the fluid at any point between the plates varies linearly between  $V = 0$  and  $V = V_B$ . See Fig. 1.5.

Let us define the velocity gradient as the change in fluid velocity with respect to  $y$ .

$$\text{Velocity gradient} \equiv dV/dy$$

The velocity profile is a graphical representation of the velocity gradient. See Fig. 1.5. For a linearly varying velocity profile like that shown in Fig. 1.5, the velocity gradient can also be written as

$$\text{Velocity gradient} = V_B/h$$

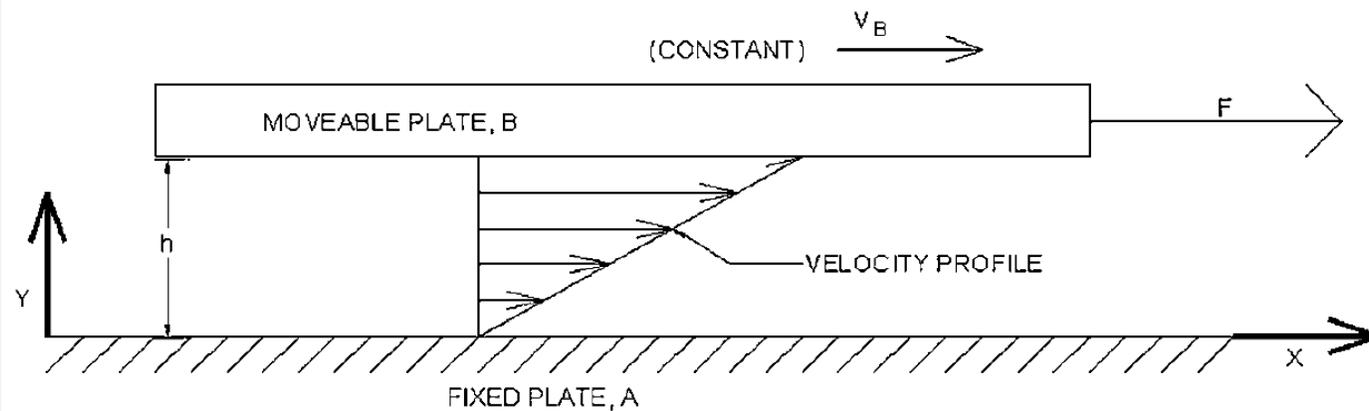


Figure 1.5 Velocity profile in a fluid between two parallel plates.

## إنحدار السرعة (Velocity gradient):

يقصد بهذا المصطلح قيمة السرعة كدالة للعمق:

$$V = f(y)$$

حيث نعتبر السرعة عند الصفيحة الثابتة تساوي صفراً، وتبدأ بالتزايد حتى تصل أعلى قيمة لها عند الصفيحة المتحركة  $V_B$ .

يكتب انحدار السرعة رياضياً بدلالة مشتقة السرعة  $V$  نسبة إلى العمق  $y$ .

يسمى الرسم البياني لانحدار السرعة بـ (توزيع السرعة - Velocity Profile).

عند دراسة الجريان بين صفيحتين متوازيتين تفصل بينهما مسافة صغيرة ( $h$ )، فإن توزيع السرعة يكون خطياً كما في الرسم أدناه.

# Understanding Viscosity: Shear Stress in a Fluid Body

Since the fluid element shown will be moving at a constant velocity, and will not be rotating, the shear stress on the element  $\tau'$  must be the same as the shear stress  $\tau$ . Therefore,

$$d\tau/dy = 0 \text{ and } \tau_A = \tau_B = \tau_{\text{wall}}$$

Physically, the shearing stress at the wall may also be represented by

$$\tau_A = \tau_B = \text{force/plate area}$$

The shear stress on a fluid is related to the rate of shearing strain. If a very large force is applied to the moving plate B, a relatively higher velocity, a higher rate of shearing strain, and a higher stress will result. In fact, the relationship between shearing stress and rate of shearing strain is determined by the fluid property known as viscosity.

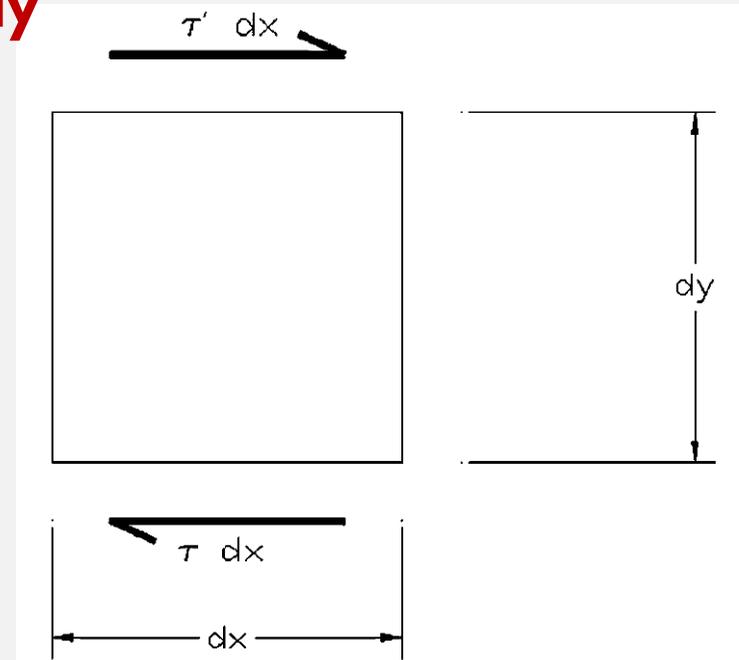


Figure 1.6 Shear stress on an element of the fluid.

**Shear Stress** is the force per unit area that acts parallel to the surface of the fluid, caused by the sliding of one layer of fluid past another. **إجهاد القص**: هو القوة المؤثرة على وحدة المساحة باتجاه موازٍ لسطح المائع، وينشأ نتيجة انزلاق طبقات المائع بعضها فوق بعض. \* هناك علاقة طردية بين **إجهاد القص  $\tau$**  في المائع و**معدل تغير انفعال القص  $\dot{\gamma}$** ، وتعتمد هذه العلاقة على خاصية المائع التي تسمى **اللزوجة  $\mu$** .

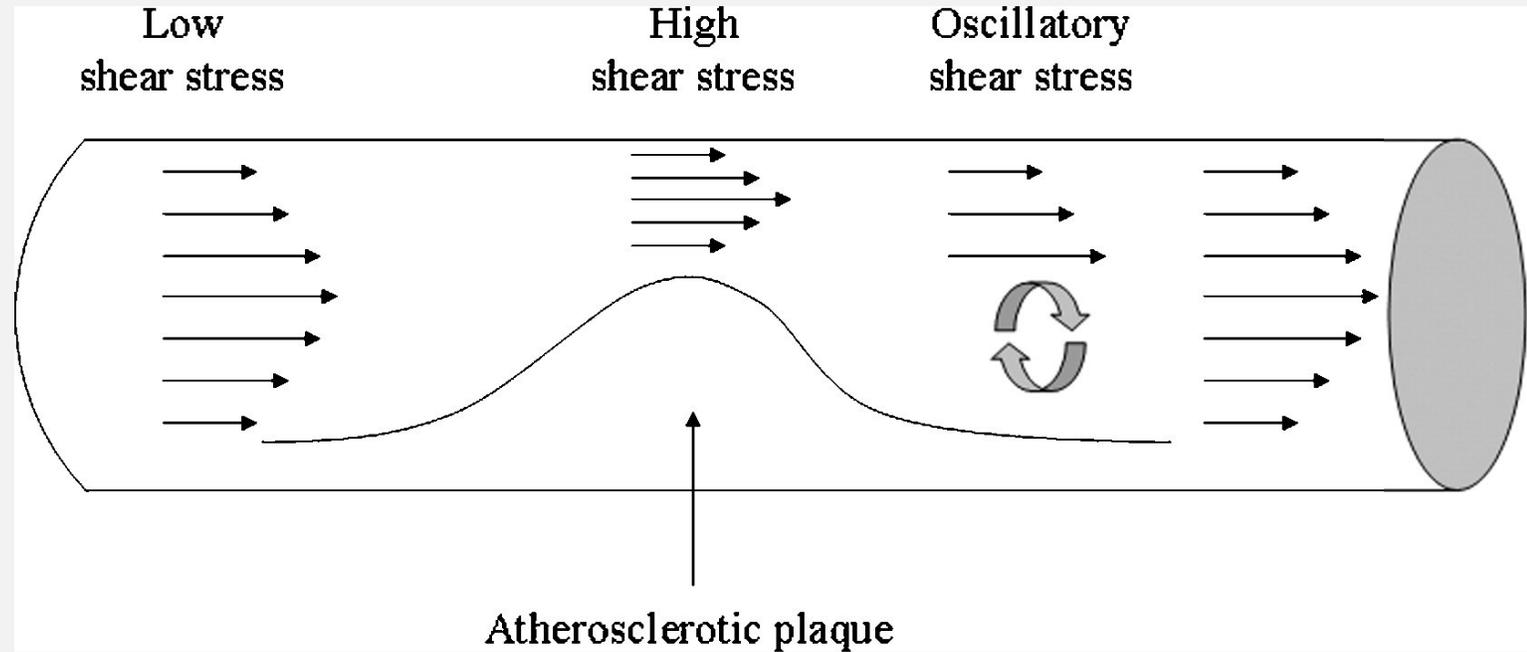
$$\tau \propto \dot{\gamma} \Rightarrow \tau = \mu \dot{\gamma} = \mu \frac{dV}{dy} = \mu \frac{V_B}{h}$$

## تأثير إجهاد القص على جريان الدم

في ميكانيكا الموائع المتعلقة بالجهاز القلبي الوعائي، يعد إجهاد القص مفهوماً شديد الأهمية. الدم سائل حي، وإذا كانت قوى القص المسلطة على هذا السائل كبيرة بما يكفي، فإن إجهاد القص الناتج يمكن أن يؤدي إلى تدمير خلايا الدم الحمراء. من ناحية أخرى، تشير الدراسات إلى وجود دور لإجهاد القص في تعديل [اللوويحات التصلبة للشرايين](#). لقد تم دراسة العلاقة بين إجهاد القص وأمراض الشرايين بشكل كبير، لكنها لم تفهم بعد بشكل جيد تماماً.

\* يمثل إجهاد القص قوة الاحتكاك التي يسببها الدم على الجدران الداخلية للأوعية الدموية (البطانة - endothelium).

In cardiovascular fluid mechanics, shear stress is a particularly important concept. Blood is a living fluid, and if the forces applied to the fluid are sufficient, the resulting shearing stress can cause red blood cells to be destroyed. On the other hand, studies indicate a role for shear stress in modulating [atherosclerotic plaques](#). The relationship between shear stress and arterial disease has been studied much, but is not yet very well understood.



### 1.2.3 Example problem: shear stress

Wall shear stress may be important in the development of various vascular disorders. For example, the shear stress of circulating blood on endothelial cells has been hypothesized to play a role in elevating vascular transport in ocular diseases such as diabetic retinopathy.

In this example problem, we are asked to estimate the wall shear stress in an arteriole in the retinal circulation. Gilmore et al. have published a related paper in the *American Journal of Physiology: Heart and Circulatory Physiology*, volume 288, in February 2005. In that article, the authors published the measured values of retinal arteriolar diameter and blood velocity in arterioles. For this problem, we will use their published values: 80  $\mu\text{m}$  for a vessel diameter and 30 mm/s for mean retinal blood flow velocity. Later in Sec. 1.4.4, we will see that, for a parabolic flow profile, a good estimate of the shearing rate is

$$\dot{\gamma} = \frac{8V_m}{D}$$

where  $V_m$  is the mean velocity across the vessel cross section and  $D$  is the vessel inside diameter.

We will also see in the next section that the shear stress is equal to the viscosity multiplied by the rate of shearing strain, that is,

$$\tau = \mu\dot{\gamma}$$

Therefore, to estimate the shear stress on the wall of a retinal arteriole, with the data from Gilmore's paper, we can calculate

$$\tau = \frac{\mu 8V_m}{D} = \frac{0.0035 \frac{\text{Ns}}{\text{m}^2} 8(3) \frac{\text{cm}}{\text{s}}}{0.008 \text{ cm}} = 10.5 \frac{\text{N}}{\text{m}^2}$$

Although 10.5 Pa seems like a low shear stress when compared to the strength of aluminum or steel, it is a relatively high shear stress when compared to a similar estimate in the aorta, 0.5 Pa. See Table 1.1.

**TABLE 1.1 Estimate of Wall Shear Stress in Various Vessels in the Human Circulatory System**

Vessel	ID, cm	$V_m$ , cm/s	Shear rate <sup>3</sup>	Shear stress, <sup>4</sup> N/m <sup>2</sup>
Aorta	2.5	48	154	0.5
Large arteriole <sup>5</sup>	0.05	1.4	224	0.8
Arteriole (retinal microcirculation <sup>6</sup> )	0.008	3	3000	10.5
Capillary	0.0008	0.7	7000	24.5

Note the increasing values for shear rate and shear stress as vessel inside diameter decreases.

## Problems

Determine the wall shear stress in a blood vessel with a diameter of **0.009 cm** if blood flows at a mean velocity of **2.8 cm/s**. Use a blood viscosity of **0.0035 N.s/m<sup>2</sup>** and the formula  $\dot{\gamma} = 8 V_m/D$ .

The first derivative of shear strain with respect to time ( $d\gamma/dt$ ) is known as \_\_\_\_\_.

- |                          |                       |
|--------------------------|-----------------------|
| (a) shear stress         | (b) velocity gradient |
| (c) rate of shear strain | (d) pressure of fluid |

The change in fluid velocity with respect to depth is known as the \_\_\_\_\_.

- |                  |                          |
|------------------|--------------------------|
| (a) viscosity    | (b) velocity gradient    |
| (c) shear stress | (d) rate of shear strain |