SAWA UNIVERSITY

ميكانيك الموائع الحيوية

Biofluid Mechanics

جامعة ساوة الأهلية

الكلية التقنية الهندسية

قسم هندسة تقنيات الفيزياء الطبية والعلاج الاشعاعي

المرحلة الثانية

السنة الدراسية: 2025 – 2026 / الفصل الأول

المحاضرة الثالثة: اجهاد القص واللزوجة - 2

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

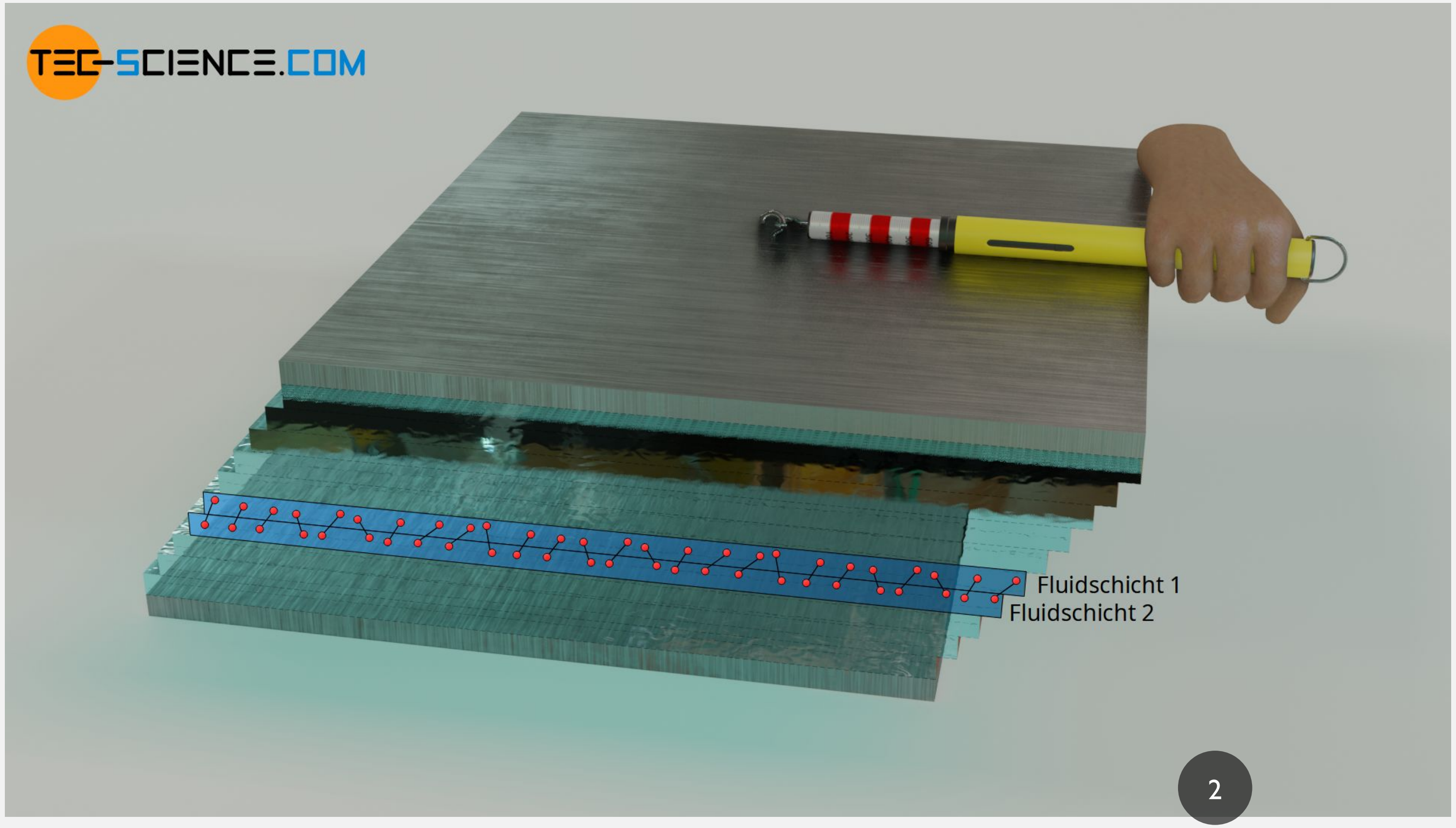
تعرَّف اللزوجة بأنها خاصية السائل التي تعطي مقاومة لحركة طبقة منه فوق طبقة أخرى مجاورة منه.

أي أن اللزوجة هي مقياس لقوة التماسك بين طبقات المائع.

In simpler terms, viscosity indicates how “thick” or “sticky” a fluid is — for example, honey has a higher viscosity than water.

بعبارة أخرى، تصف اللزوجة مقدار (ثخن) أو (دباقة) المائع – مثال ذلك أن لزوجة العسل أكبر من لزوجة الماء.

تؤثر اللزوجة على سهولة جريان (انسيابية) المائع عندما يتم سكبه



Viscosity

A common way to visualize material properties in fluids is by making a plot of shearing stress as a function of the rate of shearing strain. For the plot shown in Fig. 1.7, shearing stress is represented by the Greek character τ, and the rate of shearing strain is represented by $\dot{\gamma}$.

The material property that is represented by the slope of the stress–shearing rate curve is known as viscosity and is represented by the Greek letter μ (mu). Viscosity is also sometimes referred to by the name absolute viscosity or dynamic viscosity. For common fluids like oil, water, and air, viscosity does not vary with shearing rate. Fluids with constant viscosity are known as Newtonian fluids. For Newtonian fluids, shear stress and rate of shearing strain may be related by the following equation:

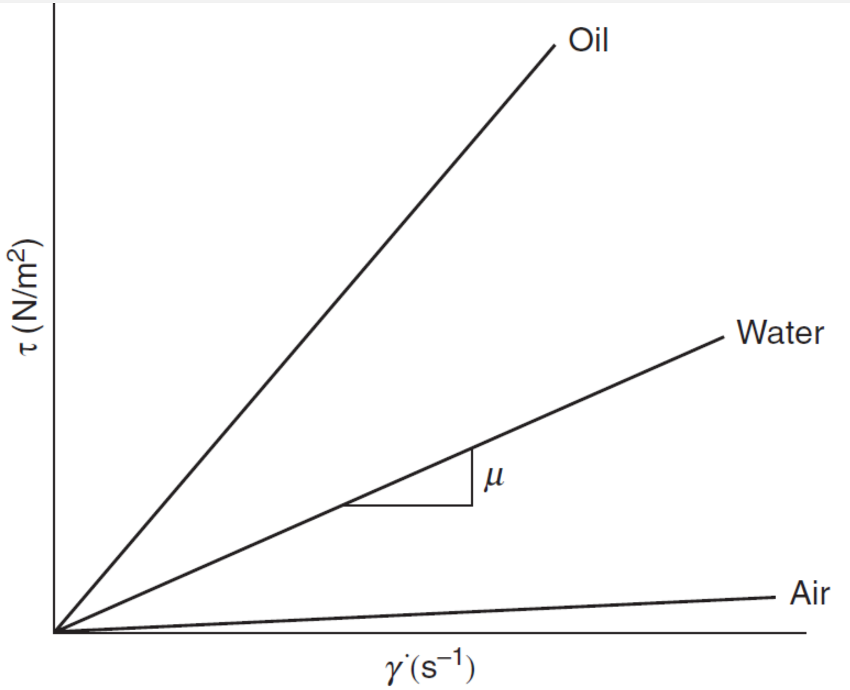


Figure 1.7 Stress versus rate of shearing strain for various fluids.

$$\tau = \mu\dot{\gamma}$$

where τ = shear stress
μ = viscosity
$\dot{\gamma}$ = the rate of shearing strain

اللزوجة μ هي ميل المنحني الناتج عن رسم إجهاد القص τ في المحور العمودي مقابل معدل تغير انفعال القص $\dot{\gamma}$ في المحور الأفقي، وتسمى أيضاً باللزوجة المطلقة او الديناميكية.

إذا كانت اللزوجة ثابتة بغض النظر عن تغير إجهاد القص فإن المائع يسمى -مائع نيوتني- وفيها تكون العلاقة بين إجهاد القص ومعدل تغير انفعال القص طردية.

Units of Viscosity

$$\mu = \frac{\tau}{\dot{\gamma}} = \frac{\tau}{\frac{dV}{dy}}$$

Unit of dynamic viscosity is **Pa.s = N.s / m^2**

Also **1.0 Pa.s = 10 Poise** (**Poise = dyne . s / cm^2**)

1 centipoise or cP = 1/100 Poise

$$\mu = \frac{\text{Shear stress}}{\frac{\text{Change of velocity}}{\text{Change of distance}}} = \frac{\text{Force/Area}}{\left(\frac{\text{Length}}{\text{Time}}\right) \times \frac{1}{\text{Length}}}$$

$$= \frac{\text{Force/(Length)}^2}{\frac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

Viscosity

Shear stress and shear rate are not linearly related for non-Newtonian fluids. Therefore, the slope of the shear stress/shear rate curve is not constant. However, we can still talk about viscosity if we define the apparent viscosity as the instantaneous slope of the shear stress/shear rate curve. See Fig. 1.8.

Shear thinning fluids are non-Newtonian fluids whose apparent viscosity decreases as shear rate increases. Latex paint is a good example of a shear thinning fluid. It is a positive characteristic of the paint that the viscosity is low when one is painting, but that the viscosity becomes higher and the paint sticks to the surface better when no shearing force is present. At low shear rates, blood is also a shear thinning fluid. However, when the shear rate increases above 100 s^{-1}, blood behaves as a Newtonian fluid.

Shear thickening fluids are non-Newtonian fluids whose apparent viscosity increases when the shear rate increases. Quicksand is a good example of a shear thickening fluid. If one tries to move slowly in quicksand, then the viscosity is low and the movement is relatively easy. If one tries to move quickly, then the viscosity increases and the movement is difficult. A mixture of cornstarch and water also forms a shear thickening non-Newtonian fluid.

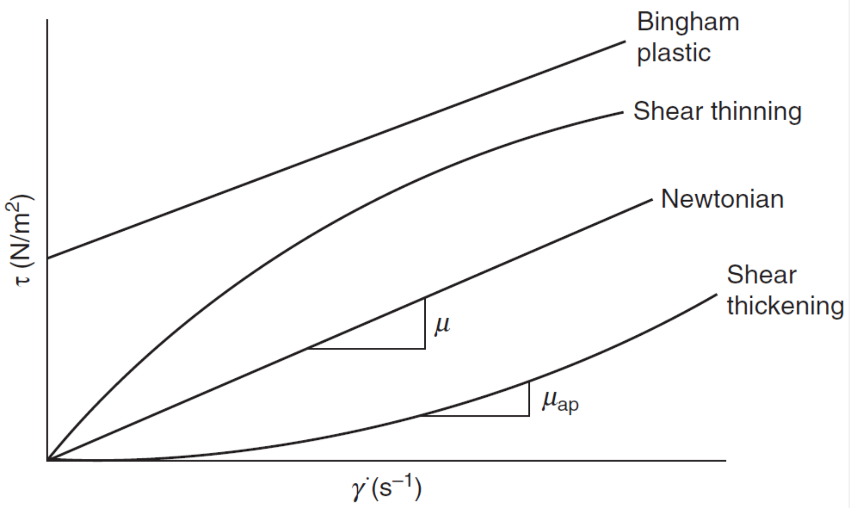


Figure 1.8 Shear stress versus rate of shearing strain for some non-Newtonian fluids.

Viscosity

A Bingham plastic is neither a fluid nor a solid. A Bingham plastic can withstand a finite shear load and flow like a fluid when that shear stress is exceeded. Toothpaste and mayonnaise are examples of Bingham plastics. Blood is also a Bingham plastic and behaves as a solid at shear rates very close to zero. The yield stress for blood is very small, approximately in the range from 0.005 to 0.01 N/m^2.

Kinematic viscosity is another fluid property that has been used to characterize flow. It is the ratio of absolute viscosity to fluid density and is represented by the Greek character ν (nu). Kinematic viscosity can be defined by the equation:

$$\nu = \frac{\mu}{\rho}$$

where μ is the absolute viscosity and ρ is the fluid density.

The SI units for absolute viscosity are Ns/m^2. The SI units for kinematic viscosity are m^2/s.

Unit of kinematic viscosity is **m^2/s**

Also **cm^2/s** is called **Stoke** (**1.0 Stoke = 10^{-4} m^2/s**)

1 centistoke = 1/100 Stoke

The units of kinematic viscosity is obtained as

$$\nu = \frac{\text{Units of } \mu}{\text{Units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \dfrac{\text{Mass}}{(\text{Length})^3}} = \frac{\text{Force} \times \text{Time}}{\dfrac{\text{Mass}}{\text{Length}}}$$

$$= \frac{\text{Mass} \times \dfrac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\left(\dfrac{\text{Mass}}{\text{Length}}\right)} \qquad \left\{\begin{aligned} \because \text{ Force} &= \text{Mass} \times \text{Acc.} \\ &= \text{Mass} \times \frac{\text{Length}}{\text{Time}^2} \end{aligned}\right\}$$

$$= \frac{(\text{Length})^2}{\text{Time}}.$$

Fundamental Method for Measuring Viscosity

A fundamental method for measuring viscosity involves a viscometer made from concentric cylinders. See Fig. 1.9. The fluid for which the viscosity is to be measured is placed between the two cylinders. The torque generated on the inner fixed cylinder by the outer rotating cylinder is determined by using a torque-measuring shaft. The force required to cause the cylinder to spin and the velocity at which it spins are also measured. Then the viscosity may be calculated in the following way:

The shear stress τ in the fluid is equal to the force F applied to the outer cylinder divided by the surface area A of the internal cylinder, that is,

$$\tau = \frac{F}{A}$$

The shear rate $\dot{\gamma}$ for the fluid in the gap, between the cylinders, may also be calculated from the velocity of the cylinder, V, and the gap width h as

$$\dot{\gamma} = \frac{V}{h}$$

From the shear stress and the shear rate, the viscosity and/or the kinematic velocity may be obtained as

$$\mu = \frac{\tau}{\dot{\gamma}} \quad \text{and} \quad \nu = \frac{\mu}{\rho}$$

where μ = viscosity
ν = kinematic viscosity
ρ = density

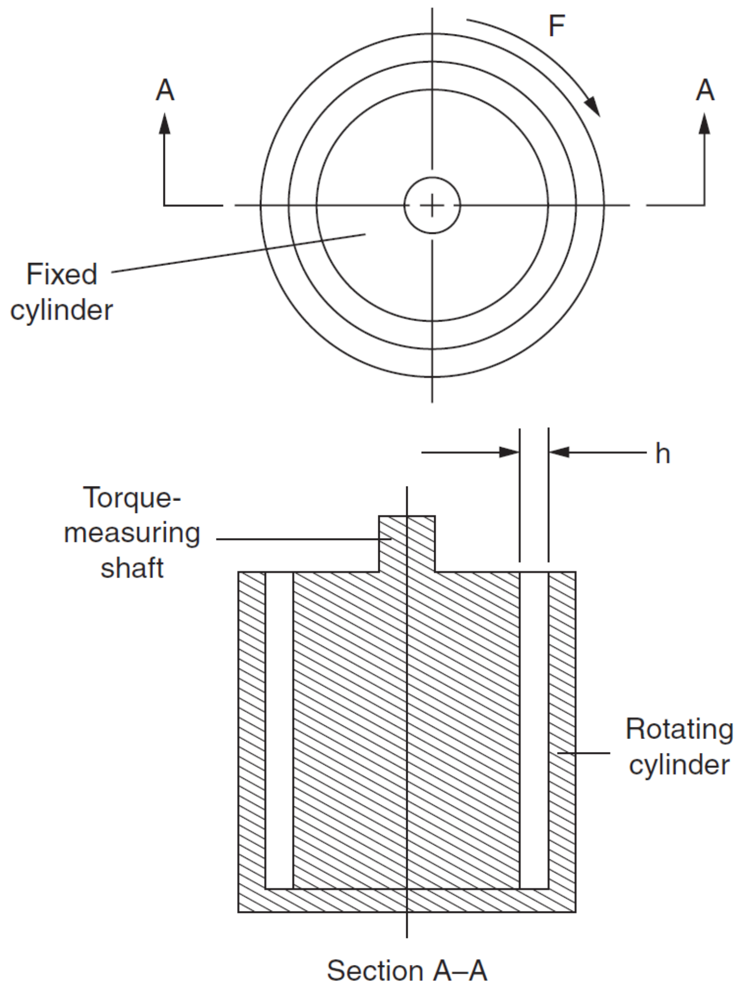


Figure 1.9*a* Cross section of a rotating cylinder viscometer.

A typical value for blood viscosity in humans is 0.0035 Ns/m^2, or 0.035 poise (P), or 3.5 cP. Note that 1 P = 1 dyne s/cm^2, or 0.1 Ns/m^2. Another useful pressure unit conversion is that 1 mmHg = 133.3 N/m^2.

Let T represent the measured torque in the viscometer shaft, and ω is its angular velocity in rad/s. Assume that D is the radius of the inner viscosimeter cylinder, and L is its length. The fluid velocity at the inner surface is

$$V = \omega \frac{D}{2}$$

It can be shown that

$$T = F \frac{D}{2}$$

leading to an equation which relates the torque, the angular velocity, and the geometric parameters of the device.

$$\mu = \frac{4T\,h}{\pi\,D^3\,L\,\omega}$$

1.3.1 Example problem: viscosity measurement

Whole blood (assume $\mu = 0.0035\ \text{Ns/m}^2$) is placed in a concentric cylinder viscometer. The gap width is 1 mm and the inner cylinder radius is 30 mm. Estimate the wall shear stress in the fluid. Assume the angular velocity of the outer cylinder to be 60 rpm.

We can begin by calculating the shear rate based on the angular velocity of the cylinder, its radius, and the gap between the inner and outer cylinders. The shear rate is equal to the velocity of the outer cylinder multiplied by the gap between the cylinders (see Fig. 1.9*a*). That is,

$$\dot{\gamma} = \frac{V}{h}$$

The wall shear stress is equal to the viscosity multiplied by the shear rate. Thus,

$$\tau = \mu\dot{\gamma} = \frac{\mu(\mathrm{r}\omega)}{h} = 0.0035\,\frac{\mathrm{Ns}}{\mathrm{m}^2}\,\frac{\left(\frac{31}{1000}\right)\mathrm{m}(60)\left(\frac{\pi}{30}\right)\frac{\mathrm{rad}}{\mathrm{s}}}{(1/1000)\mathrm{m}} = 0.682\,\frac{\mathrm{N}}{\mathrm{m}^2}$$

EXAMPLE

A blood sample is placed in a concentric cylinder viscometer. The gap width is **2 mm**, the inner cylinder radius is **42 mm**, and the angular velocity of the outer cylinder is **34 rpm**. Find the wall shear stress in the fluid (Assume the blood viscosity = **0.0035 N.s/m²**).

SOLUTION

Use Newton's law of viscosity: $\tau = \mu\dot{\gamma}$ where $\dot{\gamma}$ is the rate of shear strain in the fluid: $\dot{\gamma} = \frac{V}{h}$

V is the linear velocity of the blood particle: $V = r.\omega$ where r is the radius of motion and ω is the angular velocity.

$\omega = 34\frac{\text{round}}{\text{min}} \times 2\pi\frac{\text{rad.}}{\text{round}} = 213.6283\frac{\text{rad.}}{\text{min}}$ convert from **rpm (round/min)** to **rad/min**.

$\omega = 213.6283\frac{\text{rad.}}{\text{min}} \times \frac{1}{60\frac{\text{s}}{\text{min}}} = 3.5605\frac{\text{rad.}}{\text{s}}$ convert from **rad/min** to **rad/s**

$r = \text{radius of the inner cylinder} + \text{gap width} = \text{r}_{\text{in}} + \text{h} = 42 + 2 = \mathbf{44\ mm}$

$V = r.\omega = \frac{44}{1000}\text{m} * 3.5605\frac{\text{rad}}{\text{s}} = \mathbf{0.15666\ m/s}$

$\dot{\gamma} = \frac{V}{h} = \frac{0.15666\ \text{m/s}}{2 \times 10^{-3}\ \text{m}} = 78.33\frac{1}{\text{s}}$

$$\tau = \mu\dot{\gamma} = 0.0035\frac{\text{N.s}}{\text{m}^2} \times 78.33\frac{1}{\text{s}} = \mathbf{0.2742\frac{N}{m^2}}$$

مثال: وضعت عينة دم في جهاز قياس لزوجة ذي اسطوانتين مركزيتين عرض الفجوة بينهما 2 ملم، وقطر الاسطوانة الداخلية 42 ملم، والسرعة الزاوية لدوران الاسطوانة الخارجية 34 دورة/دقيقة. احسب اجهاد القص في عينة الدم. (افترض لزوجة الدم = 0.0035 نت.ثا/م^2).

الحل:

لإيجاد اجهاد القص نستخدم قانون نيوتن لحساب اللزوجة $\tau = \mu\dot{\gamma}$ حيث أن $\dot{\gamma}$ هي نسبة تغير انفعال القص في السائل وتحسب من القانون $\dot{\gamma} = \frac{V}{h}$

بداية نحسب قيمة السرعة الخطية V والتي تربطها العلاقة الآتية مع السرعة الزاوية (سرعة الدوران) $V = r.\omega$ حيث أن r هو **نصف قطر الحركة** أي أنه يساوي **نصف قطر الاسطوانة الداخلية + عرض الفجوة** كما في الشكل أدناه $r = r_{in} + h$

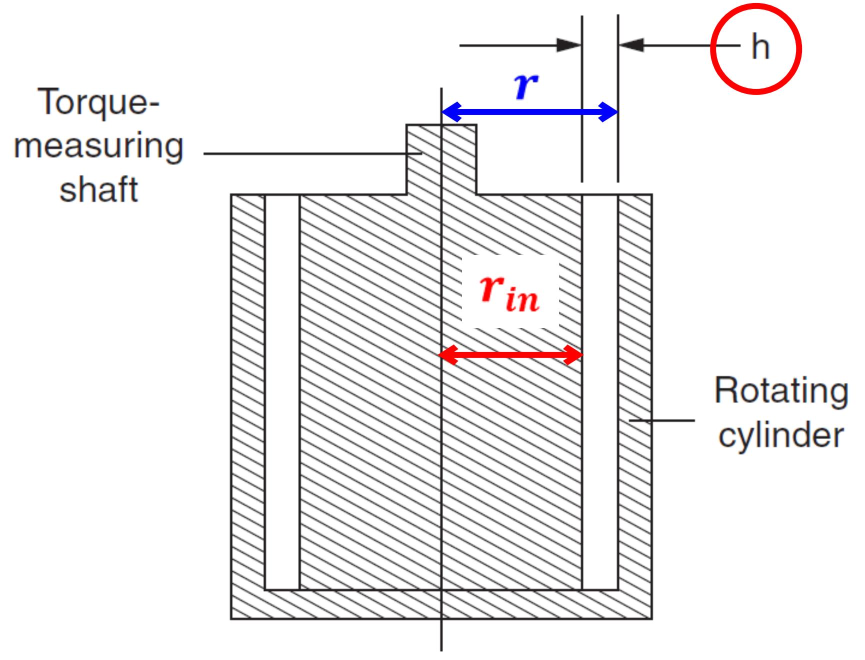


Figure 1.9a Cross section of a rotating cylinder viscometer.

- نقوم باجراء التحويلات اللازمة للسرعة الزاوية بحيث تصبح بوحدة rad/s كما موضح في الصفحة السابقة.
- لاحظ أن وحدة الدورة الكاملة (round) تعادل 2π بوحدة الزاوية نصف القطرية rad، أي أن الدورة الكاملة (round) هي الوحدة الكبيرة وتحتوي على $2\pi \approx 6.2831$ rad
- ثم نحسب السرعة الخطية من القانون $V = r.\omega$.
- ثم نحسب **نسبة تغير انفعال القص** من القانون $\dot{\gamma} = \frac{V}{h}$.

- **وأخيراً نطبق قانون نيوتن أعلاه لمعرفة إجهاد القص باستخدام لزوجة السائل المعطاة في السؤال.**

1.2.5 Clinical feature: polycythemia

Polycythemia refers to a condition in which there is an increase in hemoglobin above 17.5 g/dL in adult males or above 15.5 g/dL in females (Hoffbrand and Pettit, 1984). There is usually an icrease in the number of red blood cells above $6 \times 10^{12}\ L^{-1}$ in males and $5.5 \times 10^{12}\ L^{-1}$ in females. That is, a sufferer from this condition has a much higher blood viscosity due to this elevated red blood cell count.

Symptoms of polycythemia are typically related to an increase in blood viscosity and clotting. The symptoms include headache, dizziness, itchiness, shortness of breath, enlarged spleen, and redness in the face.

Polycythemia vera is an acquired disorder of the bone marrow that results in an increase in the number of blood cells resulting from excessive production of all three blood cell types: erythrocytes, or red blood cells; leukocytes, or white blood cells; and thrombocytes, or platelets.

The cause of polycythemia vera is not well-known. It rarely occurs in patients under 40 years. Polycythemia usually develops slowly, and a patient might not experience any problems related to the disease even after being diagnosed. In some cases, however, the abnormal bone marrow cells grow uncontrollably resulting in a type of leukemia.

In patients with polycythemia vera, there is also an increased tendency to form blood clots that can result in strokes or heart attacks. Some patients may experience abnormal bleeding because their platelets are abnormal.

The objective of the treatment is to reduce the high blood viscosity (thickness of the blood) due to the increased red blood cell mass, and to prevent hemorrhage and thrombosis.

Phlebotomy is one method used to reduce the high blood viscosity. In phlebotomy, one unit (pint) of blood is removed, weekly, until the hematocrit is less than 45; then, phlebotomy is continued as necessary. Occasionally, chemotherapy may be given to suppress the bone marrow. Other agents such as interferon may be given to lower the blood count.

A condition similar to polycythemia may be experienced by high-altitude mountaineers. Due to a combination of dehydration and excess red blood cell production brought about by extended stays at high altitudes, the climber's blood thickens dangerously. A good description of what this is like is found in the later chapters of *Annapurna,* by Maurice Herzog.