

# ميكانيك الموائع الحيوية

## Biofluid Mechanics



جامعة ساوة الأهلية

الكلية التقنية الهندسية

قسم هندسة تقنيات الفيزياء الطبية والعلاج الاشعاعي

المرحلة الثانية

السنة الدراسية: 2024 – 2025 / الفصل الأول

المحاضرة الرابعة: Fluid Statics

# FLUID STATICS

In general, fluids exert both normal and shearing forces. This section reviews a class of problems in which the fluid is at rest. A velocity gradient is necessary for the development of a shearing force. So, in the case where acceleration is equal to zero, only normal forces occur. These normal forces are also known as hydrostatic forces.

In Fig. 1.17, a point  $P_1$  in a fluid is shown at a depth of  $h$  below the surface of the fluid. The pressure exerted at a point in the fluid by the column of fluid above the point is

$$P_1 = \rho gh$$

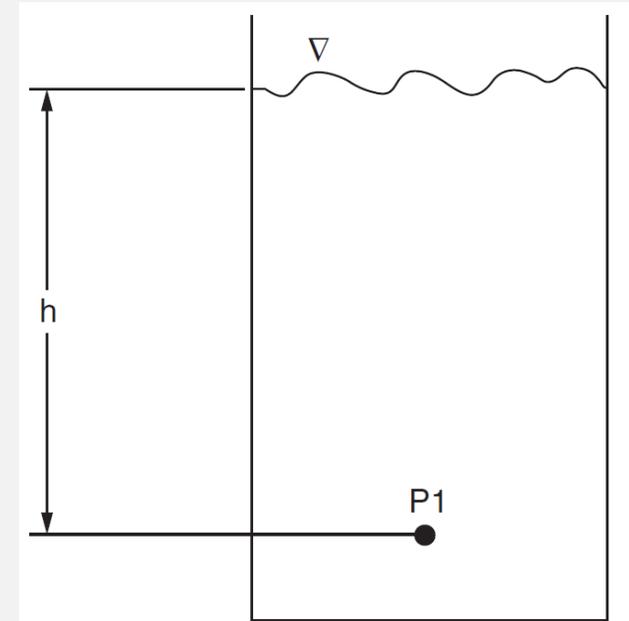
This is called the **Hydrostatic Law**.

where  $\rho$  = the fluid density

$g$  = the acceleration due to gravity

$h$  = the distance between the fluid surface and the point  $P_1$

$h$  is called the **Pressure Head** (جهد الضغط).



**Figure 1.17** Fluid in a reservoir showing the depth of point  $P_1$ .

# THE DEFINITION OF PRESSURE

**Pressure** is the amount of force applied **perpendicular** to the surface of an object per unit area. In SI units, pressure is measured by **Pascal (Pa)**, which is equal to **(N/m<sup>2</sup>)**.

A very useful unit of pressure is the “**millimeter of mercury**” mmHg (also called **Torr**).

**It is defined as the pressure exerted by a column of mercury one millimeter high** at 0°C under the acceleration due to gravity (standard gravitational acceleration of 9.80665 m/s<sup>2</sup>).

In 1954, the mmHg was set at exactly **133.322387415 Pa** as standard SI unit.

## **Reasons for this standardization:**

- **Precision and Consistency**: Physical conditions like temperature and local gravitational acceleration can introduce small variations in measurements. The redefinition eliminates these variables.
- **Safety and Practicality**: Mercury is toxic, so relying on an actual mercury column for precise measurement in laboratory or clinical settings is impractical. The redefinition allows mmHg to be used without handling mercury.

The pressure can also be expressed in **mm or cm of water**.

**Quiz:** Convert (1 mmH<sub>2</sub>O) to mmHg, and to Pa.

**Bar** is another unit of pressure:

$$1 \text{ bar} = 100 \text{ kPa} = 10^5 \text{ Pa}$$

$$1 \text{ mb (millibar)} = 0.1 \text{ kPa} = 100 \text{ Pa}$$

**Pounds per square inch (psi)** is also a commonly used unit of pressure.

$$1 \text{ psi} = 6894.76 \text{ Pa} = 51.7149 \text{ mmHg}$$

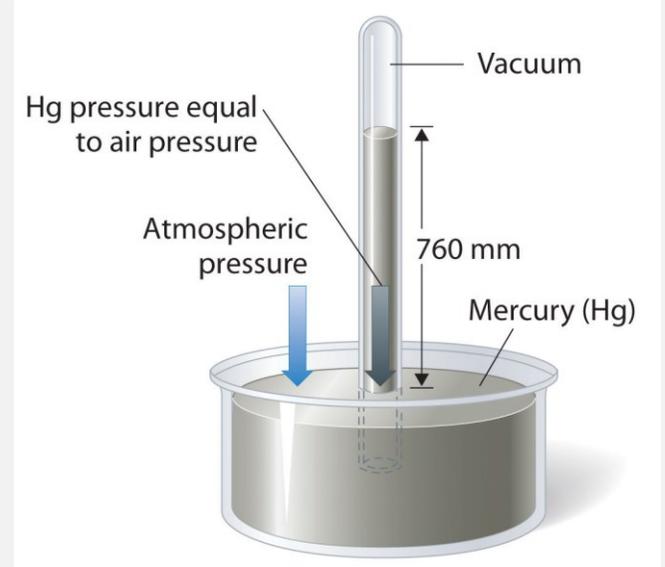
Sometimes, the unit of **kg/cm<sup>2</sup>** is used to express pressure. Here, **kg** is refers to the weight (force) that is caused by a mass of **1 kg** under the effect of gravity.

**Quiz:** Convert (1 kg/cm<sup>2</sup>) to kPa, psi, and mmHg.

**Quiz:** Convert (1 mmHg) to inHg (inches of mercury).



## Mercury Barometer



## Atmospheric Pressure (atm)

(Also known as *Air Pressure* or *Barometric Pressure*) is the pressure within the atmosphere of earth.

$$1 \text{ atm} = 101.325 \text{ kPa} = 760 \text{ mmHg} \approx 1 \text{ bar}$$

**Quiz:** Find the atmospheric pressure in *meters of water*.

## The concept of Absolute and Gauge pressures

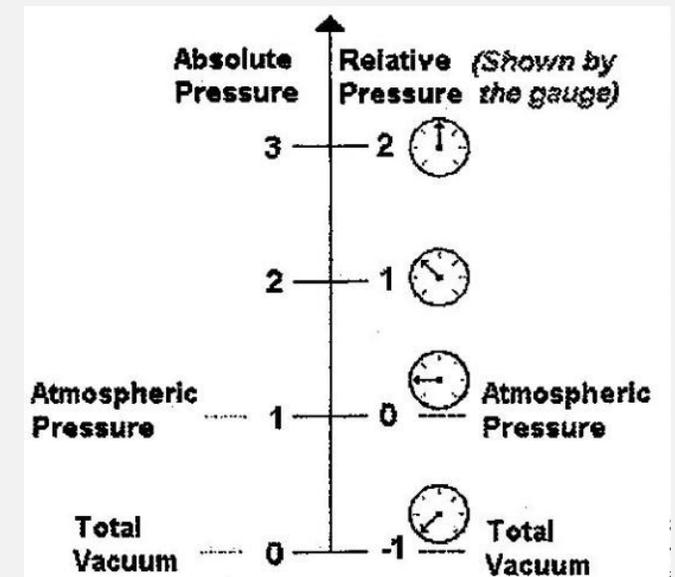
**Absolute Pressure** is the pressure measured with reference to absolute vacuum pressure.

**Gauge Pressure** is the pressure measured with reference to atmospheric pressure (i.e. **atm. pressure = zero**).

**Vacuum Pressure** is the pressure below the atmospheric pressure (i.e. **negative gauge pressure**).

$$P_{abs} = P_{atm} + P_{gauge}$$

$$P_{vac} = P_{atm} - P_{abs}$$



## Example

A hydraulic press has a ram of 30 cm diameter, and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500 N.

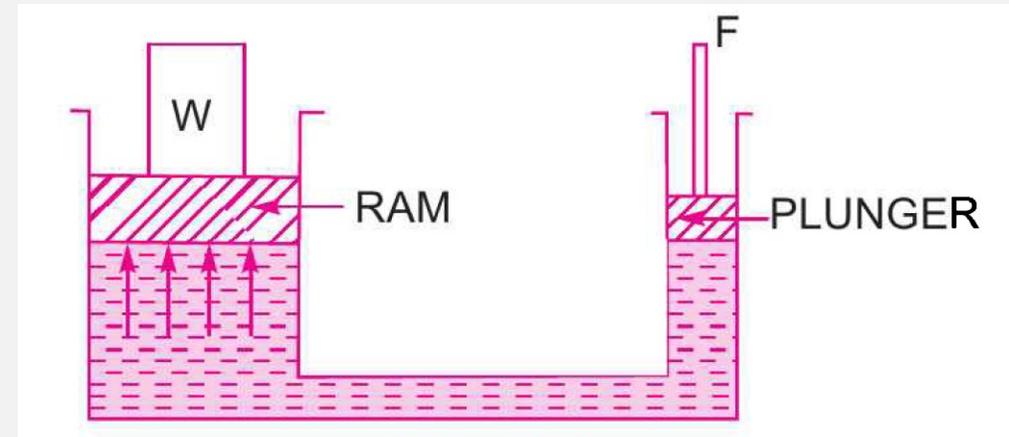
## Solution

$$P_{\text{plunger}} = P_{\text{ram}} \text{ (Pascal Law)}$$

$$P_{\text{plunger}} = \frac{F}{A_{\text{plunger}}} = \frac{500 \text{ N}}{0.00159 \text{ m}^2} = 314465.4 \text{ Pa}$$

$$P_{\text{ram}} = \frac{W}{A_{\text{ram}}} \Rightarrow 314465.4 \text{ Pa} = \frac{W}{0.07068 \text{ m}^2}$$

$$W = 22.222 \text{ kN}$$



## Example

A hydraulic press has a ram of 20 cm diameter, and a plunger of 3 cm diameter. It is used to lift a weight of 30 kN. Find the force required at the plunger.

$$\text{Ans. } F = 675.2 \text{ kN}$$

## Example

Calculate the pressure due to a column of 0.3 m of:

- water,
- an oil of specific gravity of  $S_{\text{oil}} = 0.8$ , and
- mercury of specific gravity of  $S_{\text{Hg}} = 13.6$ .

## Solution

$P = \rho gh$  (Hydrostatic Pressure)

$$(a) \quad P = \rho gh = 1000 \frac{\text{kg}}{\text{m}^3} * 9.81 \frac{\text{m}}{\text{s}^2} * 0.3 \text{ m} = 2943 \frac{\text{N}}{\text{m}^2}$$

$$(b) \quad P = \rho gh = \left(1000 \frac{\text{kg}}{\text{m}^3} * 0.8\right) * 9.81 \frac{\text{m}}{\text{s}^2} * 0.3 \text{ m} = 2354.4 \frac{\text{N}}{\text{m}^2}$$

$$(c) \quad P = \rho gh = \left(1000 \frac{\text{kg}}{\text{m}^3} * 13.6\right) * 9.81 \frac{\text{m}}{\text{s}^2} * 0.3 \text{ m} = 40024.8 \frac{\text{N}}{\text{m}^2}$$

## Example

The pressure intensity at a point in a fluid is 3.924 N/cm<sup>2</sup>. Find the corresponding height of fluid when the fluid is:

(a) water      **Ans. h = 4 m**

(b) oil of  $S = 0.9$       **Ans. h = 4.44 m**

## Remember

Specific Gravity is:

$$S = \frac{\rho_{\text{fluid}}}{\rho_{\text{water}}}$$

## Example

An oil of  $S = 0.9$  is contained in a vessel. If the height of oil at a point is 40 m, find the corresponding height of water at that point. إذا كان ارتفاع الزيت فوق النقطة يساوي 40 م فما هو الارتفاع المكافئ فيما لو استبدل الزيت بالماء؟

Ans.  $h = 36 \text{ m}$

## Example

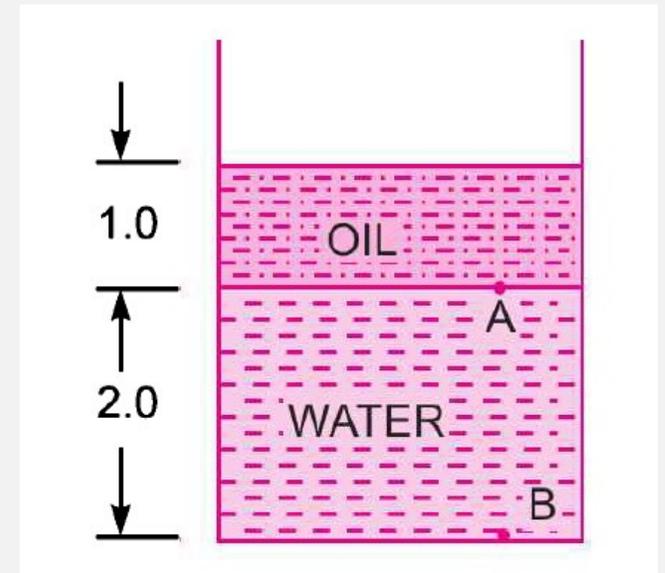
An open tank contains water up to 2.0 m height, and above it an oil ( $S = 0.9$ ) for a height of 1.0 m. Find the pressure at:

- (a) the interface of the two liquids, and
- (b) the bottom of the tank.

## Solution

$$(a) P_A = \rho_{oil} * g * h_{oil} = \left(1000 \frac{kg}{m^3} * 0.9\right) * 9.81 \frac{m}{s^2} * 1.0 \text{ m} = 8829 \frac{N}{m^2}$$

$$(b) P_B = \rho_{oil} * g * h_{oil} + \rho_{water} * g * h_{water} \\ = (1000 * 0.9) * 9.81 * 1.0 + 1000 * 9.81 * 2.0 = 28449 \frac{N}{m^2}$$



## Example

What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of  $1530 \text{ kg/m}^3$  if the atmospheric pressure is equivalent to 750 mm of mercury?

Use:  $S_{\text{Hg}} = 13.6$ .

## Solution:

### Gauge Pressure

$$P_{\text{gauge}} = \rho gh = 1530 * 9.81 * 3 = \mathbf{45027.9 \text{ Pa}}$$

### Absolute Pressure

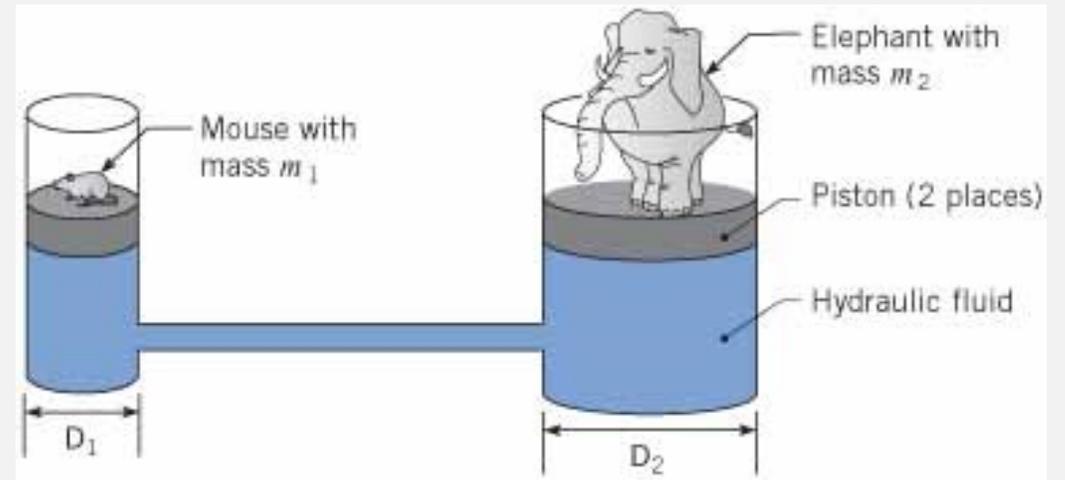
$$P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atm}}$$

$$\begin{aligned} P_{\text{atm}} &= \rho_{\text{mercury}} * g * h_{\text{mercury}} \\ &= (13.6 * 1000) * 9.81 * 750 * 10^{-3} = 100062 \text{ Pa} \end{aligned}$$

$$P_{\text{abs}} = 45027.9 + 100062 = 145089.9 \approx \mathbf{145090 \text{ Pa}}$$

## Quiz

A mouse has a mass of 25 g and an elephant a mass of 7500 kg. Determine a value of  $D_1$  and  $D_2$  so that the mouse can support the elephant.



## Quiz

For the closed tank with pressure gauges tapped into it, what is the specific gravity of the oil and pressure reading on gauge C?

