



# CONTINUOUS FUNCTION

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# CONTINUOUS FUNCTION

## The Continuity Test:

The function  $y = f(x)$  is continuous at  $x = c$  if and only if the following statements are true:

- 1-  $f(c)$  exists
  - 2-  $\lim_{x \rightarrow c} f(x)$  exists
  - 3-  $f(c) = \lim_{x \rightarrow c} f(x)$
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**Example:** did the function  $f(x) = 8 - x^3 - 2x^2$  is continuous at the  $x=2$  ?

Sol:

$$f(2) = 8 - 2^3 - 2 * (2)^2 = 8$$

$$\lim_{x \rightarrow 2} [8 - x^3 - 2x^2] = 8 - 2^3 - 2 * (2)^2 = 8$$

$$f(2) = \lim_{x \rightarrow 2} f(x).$$



*The following types of functions are continuous at every point in their domains:*

*1 – Polynomials.*

*2 – Rational functions: They have points of discontinuity at the zero of their denominators.*

*3 – Root functions: ( $y = \sqrt[n]{x}$ ,  $n$  a positive integer greater than 1).*

*4 – Trigonometric functions.*

*5 – Inverse trigonometric functions.*

*6 – Exponential functions.*

*7 – Logarithmic functions.*

*Note: The inverse function of any continuous function is continuous.*

**Example**

Show that  $f(x)$  has a continuous extension to  $x=2$  and find that extension.

$$f(x) = \frac{x^2 + x - 6}{x^2 - 4}$$

**Sol.**

Although  $f(2)$  is not defined,

If  $x \neq 2$  we have

$$f(x) = \frac{x^2 + x - 6}{x^2 - 4} = \frac{(x-2)(x+3)}{(x-2)(x+2)} = \frac{x+3}{x+2}$$

The function  $g(x) = \frac{x+3}{x+2}$  is equivalent to  $f(x) = \frac{x^2+x-6}{x^2-4}$ , but  $g(x)$  is continuous at  $x=2$  having a  $\lim_{x \rightarrow 2} g(x) = \frac{5}{4}$  and  $g(2) = 5/4$ .

Examples:

$$1) f(x) = \begin{cases} \frac{2x^2+x-3}{x-1} & x \neq 1 \\ 2 & x = 1 \end{cases}$$

$$1 - f(1) = 2$$

$$2 - \lim_{x \rightarrow 1} \frac{2x^2+x-3}{x-1} = \lim_{x \rightarrow 1} \frac{(2x+3)(x-1)}{(x-1)} = 2(1) + 3 = 5$$

$$3 - \lim_{x \rightarrow 1} f(x) \neq f(1)$$

$\therefore f(x)$  discontinuous at  $x = 1$ .

$$2) f(x) = \begin{cases} 3+x & x \leq 1 \\ 3-x & x > 1 \end{cases}$$

$$1 - f(1) = 3 + 1 = 4$$

$$2 - \lim_{x \rightarrow 1^-} 3+x = 3+1 = 4$$

$$3 - \lim_{x \rightarrow 1^+} 3-x = 3-1 = 2 \quad \therefore \lim_{x \rightarrow 1^-} 3+x \neq \lim_{x \rightarrow 1^+} 3-x$$

$\therefore f(x)$  discontinuous at  $x = 1$ .

$$3) f(x) = \begin{cases} \frac{1}{x-2} & x \neq 2 \\ 3 & x = 2 \end{cases}$$

$$f(2) = 3 \text{ \& } \lim_{x \rightarrow 2} \frac{1}{x-2} = \frac{1}{0} = \infty \quad \therefore \text{no limit, } f(x) \text{ discontinuous.}$$

$$4) f(x) = \begin{cases} \frac{x^2-16}{x-4} & x \neq 4 \\ 9 & x = 4 \end{cases}$$

$$f(4) = 9, \lim_{x \rightarrow 4} \frac{x^2-16}{x-4} = \lim_{x \rightarrow 4} \frac{(x-4)(x+4)}{(x-4)} = 8, f(4) \neq \lim_{x \rightarrow 4} f(x)$$

$\therefore f(x)$  discontinuous at  $x = 4$ .