



# POLAR COORDINATES

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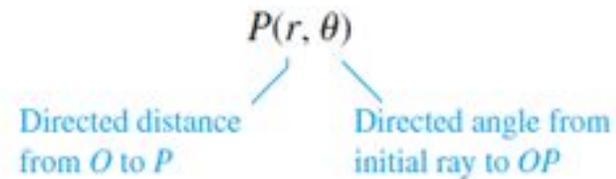
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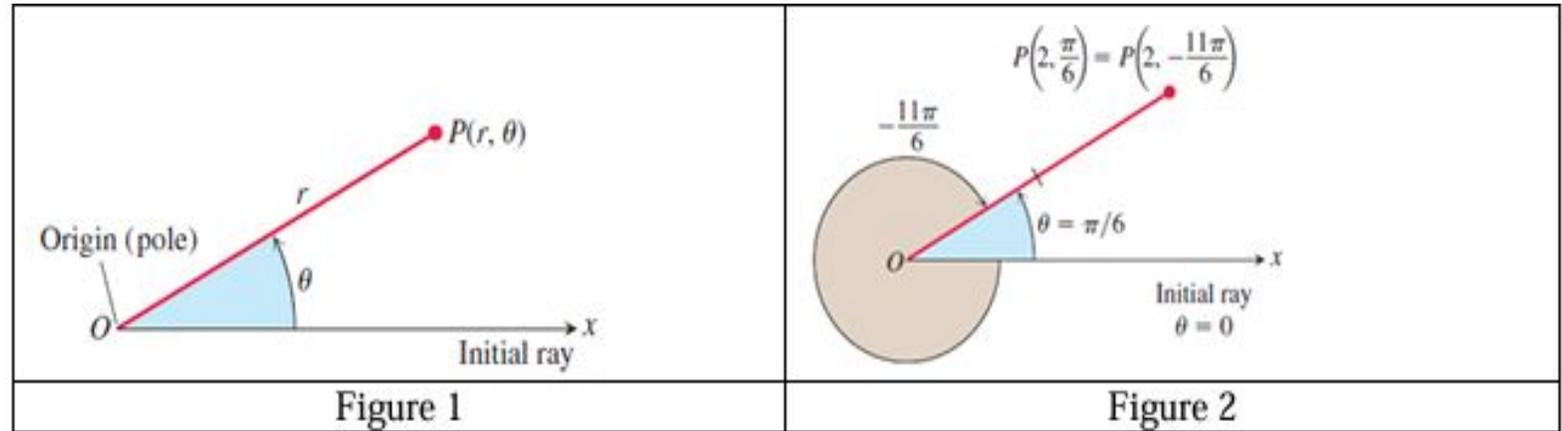
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# POLAR COORDINATES

To define polar coordinates, we first fix an origin  $O$  (called the pole) and an initial ray from  $O$  (Figure 1). Usually the positive  $x$ -axis is chosen as the initial ray. Then each point  $P$  can be located by assigning to it a polar coordinate pair  $(r, \theta)$  in which  $r$  gives the directed distance from  $O$  to  $P$  and  $\theta$  gives the directed angle from the initial ray to ray  $OP$ . So we label the point  $P$  as;



For instance, the point 2 units from the origin along the ray  $\theta = \pi/6$  has polar coordinates  $r = 2$ ,  $\theta = \pi/6$ . It also has coordinates  $r = 2$ ,  $\theta = -11\pi/6$  (Figure 2).



The usual way to relate polar and Cartesian coordinates is shown in Figure 3 and is determined using the equation below:

**Equations Relating Polar and Cartesian Coordinates**

$$x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2, \quad \tan \theta = \frac{y}{x}$$

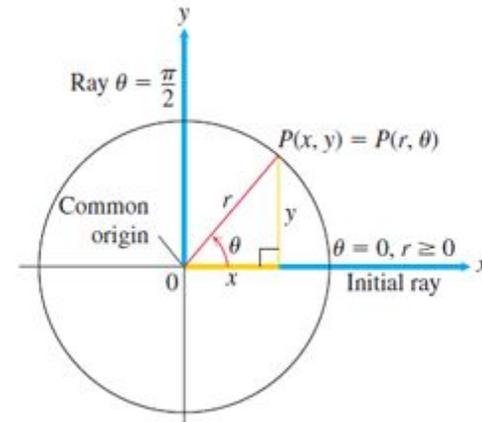
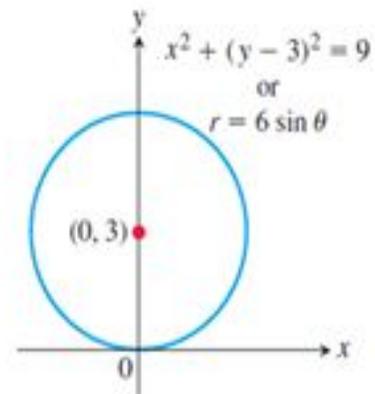


Figure 3



### EXAMPLE

Find a polar equation for the circle  $x^2 + (y - 3)^2 = 9$

$$x^2 + (y - 3)^2 = 9$$

$$x^2 + y^2 - 6y + 9 = 9$$

Expand  $(y - 3)^2$ .

$$x^2 + y^2 - 6y = 0$$

Cancelation

$$r^2 - 6r \sin \theta = 0$$

$x^2 + y^2 = r^2$ ,  $y = r \sin \theta$

$$r = 0 \quad \text{or} \quad r - 6 \sin \theta = 0$$

$$r = 6 \sin \theta$$

Includes both possibilities

**EXAMPLE:** Replace the following polar equations by equivalent Cartesian equations and identify their graphs.

(a)  $r \cos \theta = -4$

(b)  $r^2 = 4r \cos \theta$

(c)  $r = \frac{4}{2 \cos \theta - \sin \theta}$

**Solution** We use the substitutions  $r \cos \theta = x$ ,  $r \sin \theta = y$ , and  $r^2 = x^2 + y^2$ .

(a)  $r \cos \theta = -4$

The Cartesian equation:  $r \cos \theta = -4$

$$x = -4 \quad \text{Substitution}$$

The graph: Vertical line through  $x = -4$  on the  $x$ -axis

(b)  $r^2 = 4r \cos \theta$

The Cartesian equation:

$$r^2 = 4r \cos \theta$$

$$x^2 + y^2 = 4x$$

$$x^2 - 4x + y^2 = 0$$

$$x^2 - 4x + 4 + y^2 = 4$$

$$(x - 2)^2 + y^2 = 4$$

Substitution

Completing the square

Factoring

The graph: Circle, radius 2, center  $(h, k) = (2, 0)$

(c)  $r = \frac{4}{2 \cos \theta - \sin \theta}$

The Cartesian equation:

$$r(2 \cos \theta - \sin \theta) = 4$$

$$2r \cos \theta - r \sin \theta = 4$$

$$2x - y = 4$$

$$y = 2x - 4$$

Multiplying by  $r$

Substitution

Solve for  $y$ .

**EXAMPLE** Here are some plane curves expressed in terms of both polar coordinate and Cartesian coordinate equations.

Polar equation	Cartesian equivalent
$r \cos \theta = 2$	$x = 2$
$r^2 \cos \theta \sin \theta = 4$	$xy = 4$
$r^2 \cos^2 \theta - r^2 \sin^2 \theta = 1$	$x^2 - y^2 = 1$
$r = 1 + 2r \cos \theta$	$y^2 - 3x^2 - 4x - 1 = 0$
$r = 1 - \cos \theta$	$x^4 + y^4 + 2x^2y^2 + 2x^3 + 2xy^2 - y^2 = 0$