SAWA UNIVERSITY

DEFINITION OF DERIVATE

جامعة ساوة

كلية التقنية الهندسية

قسم الاجهزة الطبية

المرحلة/الاولى

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Differentiation

Definition of derivative:

The derivative is the rate of change of a function with respect to its variable, or the slope of the tangent line to the curve at a given point.

المشتقة هي معدل تغيّر دالة بالنسبة لمتغيّرها، أو هي ميل المماس للمنحنى عند نقطة معيّنة.

DEFINITION The **derivative** of the function $f(x)$ with respect to the variable x is the function f' whose value at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

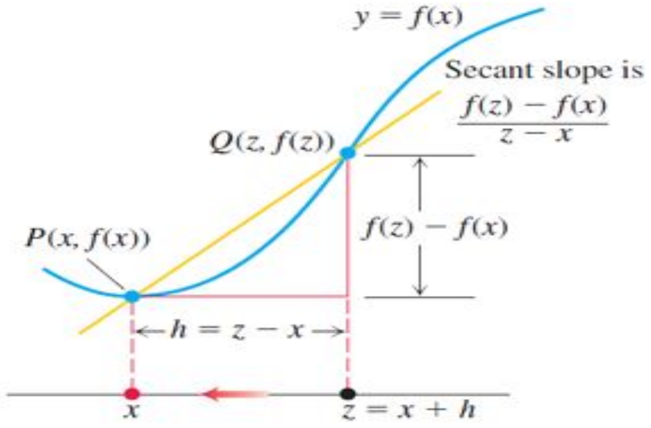
provided the limit exists.

Alternative Formula for the Derivative

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x}$$

If we write $z = x + h$, then $h = z - x$ and h approaches 0 if and only if z approaches x.

تقترب h من 0 إذا وفقط إذا اقتربت z من x.



Example 1: Find the slope of the curve y = 1/x at any point x = a ≠ 0. What is the slope at the point x = -1?

Solution:

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{\frac{1}{a+h}-\frac{1}{a}}{h} = \lim_{h\to 0}\frac{1}{h}\frac{a-(a+h)}{a(a+h)}$$

$$= \lim_{h\to 0}\frac{-h}{ha(a+h)} = \lim_{h\to 0}\frac{-1}{a(a+h)} = -\frac{1}{a^2}.$$

Example 2 : Applying the Definition, differentiate $f(x) = \frac{x}{x-1}$

Solution We use the definition of derivative, which requires us to calculate $f(x+h)$ and then subtract $f(x)$ to obtain the numerator in the difference quotient. We have

$$f(x) = \frac{x}{x-1} \quad \text{and} \quad f(x+h) = \frac{(x+h)}{(x+h)-1}, \text{ so}$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{x+h}{x+h-1}-\frac{x}{x-1}}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}\cdot\frac{(x+h)(x-1)-x(x+h-1)}{(x+h-1)(x-1)}$$

$$= \lim_{h\to 0}\frac{1}{h}\cdot\frac{-h}{(x+h-1)(x-1)}$$

$$= \lim_{h\to 0}\frac{-1}{(x+h-1)(x-1)} = \frac{-1}{(x-1)^2}.$$

Example 3:

(a) Find the derivative of $f(x) = \sqrt{x}$ for $x > 0$.

(b) Find the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.

Solution

(a) We use the alternative formula to calculate f':

$$\begin{aligned} f'(x) &= \lim_{z \to x} \frac{f(z) - f(x)}{z - x} \\ &= \lim_{z \to x} \frac{\sqrt{z} - \sqrt{x}}{z - x} \\ &= \lim_{z \to x} \frac{\sqrt{z} - \sqrt{x}}{(\sqrt{z} - \sqrt{x})(\sqrt{z} + \sqrt{x})} \qquad \frac{1}{a^2 - b^2} = \frac{1}{(a - b)(a + b)} \\ &= \lim_{z \to x} \frac{1}{\sqrt{z} + \sqrt{x}} = \frac{1}{2\sqrt{x}}. \qquad \text{Cancel and evaluate.} \end{aligned}$$

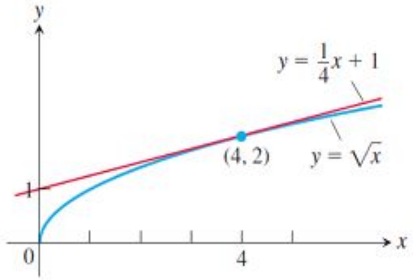
(b) The slope of the curve at $x = 4$ is

$$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}.$$

The tangent is the line through the point (4, 2) with slope 1/4 (Figure 3.5):

$$y = 2 + \frac{1}{4}(x - 4)$$

$$y = \frac{1}{4}x + 1.$$



Differentiation Rules:

If f(x) or y is differentiable function of X and C is constant then:

1- $\frac{d}{dx}(c) = 0$

2- $\frac{d}{dx}(c\,f(x)) = c\,\frac{d\,f(x)}{dx}$

3- $\frac{d}{dx}(f(x) \mp g(x)) = \frac{d\,f(x)}{dx} \mp \frac{d\,g(x)}{dx} = f'(x) \mp g'(x)$

4- $\frac{d}{dx}(f(x)\,g(x)) = f(x)\,g'(x) + g(x)f'(x)$

5- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)\,g'(x)}{(g(x))^2}$

6- $\frac{d}{dx}(x^n) = n\,x^{n-1}$

7- $\frac{d}{dx}(f(x))^n = n\,((f(x))^{n-1}\,.f'(x)$

Example 4:

Differentiate the following powers of x:

(a) x^3 **(b)** $x^{2/3}$ **(c)** $x^{\sqrt{2}}$ **(d)** $\frac{1}{x^4}$

Solution

(a) $\frac{d}{dx}(x^3) = 3x^{3-1} = 3x^2$

(b) $\frac{d}{dx}(x^{2/3}) = \frac{2}{3}x^{(2/3)-1} = \frac{2}{3}x^{-1/3}$

(c) $\frac{d}{dx}(x^{\sqrt{2}}) = \sqrt{2}x^{\sqrt{2}-1}$

(d) $\frac{d}{dx}\left(\frac{1}{x^4}\right) = \frac{d}{dx}(x^{-4}) = -4x^{-4-1} = -4x^{-5} = -\frac{4}{x^5}$

Example 5: Find the derivative of the functions :

1- $y = x^3 + \frac{4}{3}x^2 - 5x + 1$

$$\frac{dy}{dx} = \frac{d}{dx}x^3 + \frac{d}{dx}\left(\frac{4}{3}x^2\right) - \frac{d}{dx}(5x) + \frac{d}{dx}(1)$$

$$= 3x^2 + \frac{4}{3}\cdot 2x - 5 + 0 = 3x^2 + \frac{8}{3}x - 5$$

2- $f(x) = x^7(x^5 + x^2)$

$f'(x) = x^7(5x^4 + 2x) + 7x^6(x^5 + x^2)$

3- $f(x) = \frac{5}{x}$, $f(x) = 5x^{-1}$, $f'(x) = -5x^{-2} = -\frac{5}{x^2}$

4- $f(x) = \frac{3x^2 - 5x}{7}$, $f'(x) = \frac{1}{7}(6x - 5)$