SAWA UNIVERSITY

DEFINITION OF DERIVATIVE

جامعة ساوة

كلية التقنية الهندسية

قسم الاجهزة الطبية

المرحلة/الاولى

اسم المحاضر/م.م مريم علي سامي

Second- and Higher-Order Derivatives

If $y = f(x)$ is a differentiable function, then its derivative $f'(x)$ is also a function. If f' is also differentiable, then we can differentiate f' to get a new function of x denoted by f''. So $f''= (f')'$. The function f'' is called the **second derivative** of f because it is the derivative of the first derivative.

إذا كانت الدالة $y = f(x)$ قابلة للاشتقاق، فإن مشتقتها $f'(x)$ هي أيضًا دالة. وإذا كانت f' قابلة للاشتقاق أيضًا، فيمكننا اشتقاق f' للحصول على دالة جديدة لـ x نرمز لها بـ f''. إذن $(f')' = f''$. تُسمى الدالة f'' بالمشتقة الثانية للدالة f لأنها مشتقة المشتقة الأولى.

It is written in several ways:

It is written in several ways:

$$f''(x) = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dy'}{dx} = y'' = D^2(f)(x) = D_x^2 f(x).$$

If $y = x^6$, then $y' = 6x^5$ and we have

$$y'' = \frac{dy'}{dx} = \frac{d}{dx}(6x^5) = 30x^4.$$

If y'' is differentiable, its derivative, $y''' = dy''/dx = d^3y/dx^3$, is the **third derivative** of y with respect to x.

The names continue as you imagine, with $y^{(n)} = \frac{d}{dx}y^{(n-1)} = \frac{d^ny}{dx^n} = D^ny$

denoting t

he ***n*th derivative** of y with respect to x for any positive integer n.

Derivatives of Trigonometric Functions

Because sin x and cos x are differentiable functions of x, the related functions

$$\tan x = \frac{\sin x}{\cos x}, \qquad \cot x = \frac{\cos x}{\sin x}, \qquad \sec x = \frac{1}{\cos x}, \qquad \text{and} \qquad \csc x = \frac{1}{\sin x}$$

1- $\frac{d}{dx}(\sin u) = \cos u.\frac{du}{dx}$

2- $\frac{d}{dx}(\cos u) = -\sin u.\frac{du}{dx}$

3- $\frac{d}{dx}(\tan u) = \sec^2 u.\frac{du}{dx}$

4- $\frac{d}{dx}(\cot u) = -\csc^2 u.\frac{du}{dx}$

5- $\frac{d}{dx}(\sec u) = \sec u \tan u.\frac{du}{dx}$

6- $\frac{d}{dx}(\csc u) = -\csc u \cot u.\frac{du}{dx}$

Example 6:

The first four derivatives of $y = x^3 - 3x^2 + 2$ are

First derivative: $y' = 3x^2 - 6x$

Second derivative: $y'' = 6x - 6$

Third derivative: $y''' = 6$

Fourth derivative: $y^{(4)} = 0.$

Example 7 :

a) $y = x^2 \sin x$: $\quad \dfrac{dy}{dx} = x^2 \dfrac{d}{dx}(\sin x) + 2x \sin x$

$$= x^2 \cos x + 2x \sin x.$$

b) $y = \dfrac{\cos x}{1 - \sin x}$:

$$\frac{dy}{dx} = \frac{(1 - \sin x)\dfrac{d}{dx}(\cos x) - \cos x \dfrac{d}{dx}(1 - \sin x)}{(1 - \sin x)^2} \qquad \text{Quotient Rule}$$

$$= \frac{(1 - \sin x)(-\sin x) - \cos x(0 - \cos x)}{(1 - \sin x)^2}$$

$$= \frac{1 - \sin x}{(1 - \sin x)^2} \qquad \sin^2 x + \cos^2 x = 1$$

$$= \frac{1}{1 - \sin x}$$

c) Find y'' if $y = \sec x$.

$$y = \sec x$$

$$y' = \sec x \tan x$$

$$y'' = \frac{d}{dx}(\sec x \tan x)$$

$$= \sec x \frac{d}{dx}(\tan x) + \tan x \frac{d}{dx}(\sec x)$$

$$= \sec x (\sec^2 x) + \tan x (\sec x \tan x)$$

$$= \sec^3 x + \sec x \tan^2 x$$

d) $y = \sin\,(x^5 - x\,)$

$y' = [\,\cos\,(x^5 - x\,)].\,(5\,x^4 - 1\,)$

e) $y = \cos\left(\frac{x}{x+2}\right)$

$y' = -\sin\left(\frac{x}{x+2}\right)\left(\frac{(x+2)-x}{(x+2)^2}\right) = -\left(\frac{2}{(x+2)^2}\right)\sin\left(\frac{x}{x+2}\right)$

f) $y = \sin^5 x = (\,\sin x\,)^5$

$y' = 5\,(\sin x\,)^4\,.\,\cos x = 5\cos x\,\sin^4 x$