SAWA UNIVERSITY

DERIVATIVE

جامعة ساوة

كلية التقنية الهندسية

قسم الاجهزة الطبية

المرحلة/الاولى

اسم المحاضر/م.م مريم علي سامي

DERIVATIVE

The derivative of a function represents the rate of change of one variable with respect to another variable

DEFINITION The **derivative of a function f at a point x_0**, denoted $f'(x_0)$, is

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

provided this limit exists.

Differentiation Rules

1- Derivative of a Constant Function If f has the constant value $f(x) = c$, then,

$$\frac{dy}{dx} = \frac{d}{dx}(c) = 0$$

EXAMPLE: Find the derivative of (a) $f(x)=7$ b) $f(x)=-32$ (c) $f(x)=4/7$

Solution:

(a) $\frac{d}{dx}(7) = 0$

(b) $\frac{d}{dx}(-32) = 0$

(c) $\frac{d}{dx}(4/7) = 0$

2- Derivative of a Positive Integer Power

If n is a positive integer, then,

$$\frac{d}{dx}x^n = nx^{n-1}$$

EXAMPLE Differentiate the following powers of x.

(a) x^3 **(b)** $x^{2/3}$ **(c)** $x^{\sqrt{2}}$ **(d)** $\frac{1}{x^4}$ **(e)** $x^{-4/3}$ **(f)** $\sqrt{x^{2+\pi}}$

Solution

(a) $\frac{d}{dx}(x^3) = 3x^{3-1} = 3x^2$

(b) $\frac{d}{dx}(x^{2/3}) = \frac{2}{3}x^{(2/3)-1} = \frac{2}{3}x^{-1/3}$

(c) $\frac{d}{dx}(x^{\sqrt{2}}) = \sqrt{2}x^{\sqrt{2}-1}$

(d) $\frac{d}{dx}\left(\frac{1}{x^4}\right) = \frac{d}{dx}(x^{-4}) = -4x^{-4-1} = -4x^{-5} = -\frac{4}{x^5}$

(e) $\frac{d}{dx}(x^{-4/3}) = -\frac{4}{3}x^{-(4/3)-1} = -\frac{4}{3}x^{-7/3}$

(f) $\frac{d}{dx}\left(\sqrt{x^{2+\pi}}\right) = \frac{d}{dx}\left(x^{1+(\pi/2)}\right) = \left(1 + \frac{\pi}{2}\right)x^{1+(\pi/2)-1} = \frac{1}{2}(2 + \pi)\sqrt{x^{\pi}}$

3- Derivative Constant Multiple Rule

If u is a differentiable function of x, and c is a constant, then,

$$\frac{d}{dx}(cu) = c\frac{du}{dx}$$

EXAMPLE:

$$\frac{d}{dx}(3x^2) = 3 \cdot 2x = 6x$$

$$\frac{d}{dx}(-u) = \frac{d}{dx}(-1 \cdot u) = -1 \cdot \frac{d}{dx}(u) = -\frac{du}{dx}.$$

4- Derivative Sum Rule

If u and y are differentiable functions of x, then their sum $u + y$ is differentiable at every point where u and y are both differentiable. At such points,

$$\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$$

EXAMPLE: If $y=x^4+12x$, find dy/dx

Solution:

$$\frac{dy}{dx} = \frac{d}{dx}(x^4) + \frac{d}{dx}(12x) = 4x^3 + 12$$

7- Derivative Product Rule

If u and v are differentiable at x, then so is their product uv, and,

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

EXAMPLE Find the derivative of **(a)** $y = \frac{1}{x}(x^2 + e^x)$, **(b)** $y = e^{2x}$.

Solution

(a) We apply the Product Rule with $u = 1/x$ and $v = x^2 + e^x$:

$$\frac{d}{dx}\left[\frac{1}{x}(x^2 + e^x)\right] = \frac{1}{x}(2x + e^x) + (x^2 + e^x)\left(-\frac{1}{x^2}\right)$$

$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$, and $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

$$= 2 + \frac{e^x}{x} - 1 - \frac{e^x}{x^2}$$

$$= 1 + (x - 1)\frac{e^x}{x^2}.$$

(b) $\frac{d}{dx}(e^{2x}) = \frac{d}{dx}(e^x \cdot e^x) = e^x \cdot \frac{d}{dx}(e^x) + e^x \cdot \frac{d}{dx}(e^x) = 2e^x \cdot e^x = 2e^{2x}$

EXAMPLE Find the derivative of $y = (x^2 + 1)(x^3 + 3)$.

Solution

(a) From the Product Rule with $u = x^2 + 1$ and $v = x^3 + 3$, we find

$$\frac{d}{dx}\left[(x^2 + 1)(x^3 + 3)\right] = (x^2 + 1)(3x^2) + (x^3 + 3)(2x)$$

$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

$$= 3x^4 + 3x^2 + 2x^4 + 6x$$

$$= 5x^4 + 3x^2 + 6x.$$

EXAMPLE Find the derivative of the polynomial $y = x^3 + \frac{4}{3}x^2 - 5x + 1$.

Solution $\frac{dy}{dx} = \frac{d}{dx}x^3 + \frac{d}{dx}\left(\frac{4}{3}x^2\right) - \frac{d}{dx}(5x) + \frac{d}{dx}(1)$ Sum and Difference Rules

$$= 3x^2 + \frac{4}{3}\cdot 2x - 5 + 0 = 3x^2 + \frac{8}{3}x - 5$$

5- Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

6- Derivative of the Natural Log Function

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

8- Derivative Quotient Rule

If u and v are differentiable at x and if $u(x) \neq 0$, then the quotient u/v is differentiable at x, and,

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

EXAMPLE Find the derivative of **(a)** $y = \frac{t^2 - 1}{t^3 + 1}$, **(b)** $y = e^{-x}$.

Solution

(a) We apply the Quotient Rule with $u = t^2 - 1$ and $v = t^3 + 1$:

$$\frac{dy}{dt} = \frac{(t^3 + 1) \cdot 2t - (t^2 - 1) \cdot 3t^2}{(t^3 + 1)^2} \qquad \frac{d}{dt}\left(\frac{u}{v}\right) = \frac{v(du/dt) - u(dv/dt)}{v^2}$$

$$= \frac{2t^4 + 2t - 3t^4 + 3t^2}{(t^3 + 1)^2}$$

$$= \frac{-t^4 + 3t^2 + 2t}{(t^3 + 1)^2}.$$

9- Derivatives of Trigonometric Functions

A. $\frac{d}{dx}(\sin x) = \cos x$

B. $\frac{d}{dx}(\cos x) = -\sin x$

C. $\frac{d}{dx}(\tan x) = \sec^2 x$

D. $\frac{d}{dx}(\cot x) = -\csc^2 x$

E. $\frac{d}{dx}(\sec x) = \sec x \tan x$

F. $\frac{d}{dx}(\csc x) = -\csc x \cot x$

10- Derivatives of Inverse Trigonometric Functions, Hyperbolic Functions, and Inverse Hyperbolic Functions

Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2} \qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1 + x^2} \qquad \frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2 - 1}}$$

Hyperbolic Functions

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x \qquad \frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x \qquad \frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

Inverse Hyperbolic Functions

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1 + x^2}} \qquad \frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1 - x^2} \qquad \frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1 - x^2}}$$

$$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1 - x^2} \qquad \frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{1 + x^2}}$$

11. The Chain Rule

A. Definition: Suppose that $f \circ g$ is the composite of two differentiable function $y=f(u)$ and $u=g(x)$. Then $f \circ g$ is a differentiable function of x whose derivative at each value of x is:

$$\text{or } \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \quad (f \circ g)'(x) = f'[g(x)] \cdot g'(x)$$

B. Generalized Formulas: Let u be a differentiable function of x,

1. $\frac{d}{dx}(u^r) = ru^{r-1}\frac{du}{dx}$

2. $\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$

3. $\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$

4. $\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx}$

5. $\frac{d}{dx}(\cot u) = -\csc^2 u \frac{du}{dx}$

6. $\frac{d}{dx}(\sec u) = \sec u \tan u \frac{du}{dx}$

7. $\frac{d}{dx}(\csc u) = -\csc u \cot u \frac{du}{dx}$