SAWA UNIVERSITY

DERIVATIVE

جامعة ساوة

كلية التقنية الهندسية

قسم الاجهزة الطبية

المرحلة/الاولى

اسم المحاضر/م.م مريم علي سامي

EXAMPLE The function

$$y = (3x^2 + 1)^2$$

is the composite of $y = f(u) = u^2$ and $u = g(x) = 3x^2 + 1$. Calculating derivatives, we see that

$$\begin{aligned} \frac{dy}{du} \cdot \frac{du}{dx} &= 2u \cdot 6x \\ &= 2(3x^2 + 1) \cdot 6x \qquad \text{Substitute for } u \\ &= 36x^3 + 12x. \end{aligned}$$

Calculating the derivative from the expanded formula $(3x^2 + 1)^2 = 9x^4 + 6x^2 + 1$ gives the same result:

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(9x^4 + 6x^2 + 1) \\ &= 36x^3 + 12x. \end{aligned}$$

The derivative of the composite function $f(g(x))$ at x is the derivative of f at $g(x)$ times the derivative of g at x. This is known as the Chain Rule .

EXAMPLE Differentiate $\sin(x^2 + e^x)$ with respect to x.

Solution We apply the Chain Rule directly and find

$$\frac{d}{dx}\sin\underbrace{(x^2 + e^x)}_{\text{inside}} = \cos\underbrace{(x^2 + e^x)}_{\text{inside left alone}} \cdot \underbrace{(2x + e^x)}_{\text{derivative of the inside}}.$$

EXAMPLE Differentiate $y = e^{\cos x}$.

Solution Here the inside function is $u = g(x) = \cos x$ and the outside function is the exponential function $f(x) = e^x$. Applying the Chain Rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}(e^{\cos x}) = e^{\cos x}\frac{d}{dx}(\cos x) = e^{\cos x}(-\sin x) = -e^{\cos x}\sin x.$$

we see that the Chain Rule gives the formula

$$\frac{d}{dx}e^u = e^u\frac{du}{dx}.$$

EXAMPLE

$$\frac{d}{dx}\left(e^{x^2}\right) = e^{x^2}\cdot\frac{d}{dx}(x^2) = 2xe^{x^2}.$$

EXAMPLE Find the derivative of $g(t) = \tan(5 - \sin 2t)$.

Solution Notice here that the tangent is a function of $5 - \sin 2t$, whereas the sine is a function of $2t$, which is itself a function of t. Therefore, by the Chain Rule,

$$\begin{aligned} g'(t) &= \frac{d}{dt}(\tan(5 - \sin 2t)) \\ &= \sec^2(5 - \sin 2t)\cdot\frac{d}{dt}(5 - \sin 2t) && \text{Derivative of } \tan u \text{ with } u = 5 - \sin 2t \\ &= \sec^2(5 - \sin 2t)\cdot\left(0 - \cos 2t\cdot\frac{d}{dt}(2t)\right) && \text{Derivative of } 5 - \sin u \text{ with } u = 2t \\ &= \sec^2(5 - \sin 2t)\cdot(-\cos 2t)\cdot 2 \\ &= -2(\cos 2t)\sec^2(5 - \sin 2t). \end{aligned}$$

EXAMPLE

$$\frac{d}{dx}(5x^3 - x^4)^7 = 7(5x^3 - x^4)^6 \frac{d}{dx}(5x^3 - x^4)$$

Power Chain Rule with $u = 5x^3 - x^4, n = 7$

$$= 7(5x^3 - x^4)^6(5 \cdot 3x^2 - 4x^3)$$
$$= 7(5x^3 - x^4)^6(15x^2 - 4x^3)$$

EXAMPLE

$$\frac{d}{dx}\left(\frac{1}{3x - 2}\right) = \frac{d}{dx}(3x - 2)^{-1}$$
$$= -1(3x - 2)^{-2}\frac{d}{dx}(3x - 2)$$

Power Chain Rule with $u = 3x - 2, n = -1$

$$= -1(3x - 2)^{-2}(3)$$
$$= -\frac{3}{(3x - 2)^2}$$

EXAMPLE

$$\frac{d}{dx}(\sin^5 x) = 5\sin^4 x \cdot \frac{d}{dx}\sin x$$
$$= 5\sin^4 x \cos x$$

Power Chain Rule with $u = \sin x, n = 5$, because $\sin^n x$ means $(\sin x)^n, n \neq -1$.

EXAMPLE

$$\frac{d}{dx}\ln(x^2 + 3) = \frac{1}{x^2 + 3}\cdot\frac{d}{dx}(x^2 + 3) = \frac{1}{x^2 + 3}\cdot 2x = \frac{2x}{x^2 + 3}.$$

12. Implicit Differentiation

A. Procedure: Given an equation involving x and y, and assuming y is a differentiable function of x, we can find $\frac{dy}{dx}$ as follows:

1. Differentiate both sides of the equation with respect to x.

2. Collect all terms involving $\frac{dy}{dx}$ on the left side of the equation, and move all other terms to the right side of the equation.

3. Factor $\frac{dy}{dx}$ out of the left side of the equation.

4. Solve for $\frac{dy}{dx}$ by dividing both sides of the equation by the left-hand factor that does not contain $\frac{dy}{dx}$.

EXAMPLE: Find dy/dx if $y^2=x$

Solution:

$$2y\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2y}$$

EXAMPLE

$$x^3 + y^3 - 9xy = 0$$

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) - \frac{d}{dx}(9xy) = \frac{d}{dx}(0)$$

$$3x^2 + 3y^2\frac{dy}{dx} - 9\left(x\frac{dy}{dx} + y\frac{dx}{dx}\right) = 0$$

Differentiate both sides with respect to x.

$$(3y^2 - 9x)\frac{dy}{dx} + 3x^2 - 9y = 0$$

Treat xy as a product and y as a function of x.

$$3(y^2 - 3x)\frac{dy}{dx} = 9y - 3x^2$$

$$\frac{dy}{dx} = \frac{3y - x^2}{y^2 - 3x}.$$

Solve for dy/dx.

EXAMPLE: If $y=3x^2+15x-3$, find d^2y/dx^2

Solution:

$$\frac{dy}{dx} = 6x + 15$$

$$\frac{d^2y}{dx^2} = 6$$

EXAMPLE: If $y=2/x^2$, find d^2y/dx^2

Solution: $\frac{dy}{dx} = 2(-2x^{-3}) = -4x^{-3}$

$$\frac{d^2y}{dx^2} = -4(-3x^{-4}) = 12x^{-4} = \frac{12}{x^4}$$

EXAMPLE: If $y = \sin^2 2x$, find d^2y/dx^2

Solution:

$$\frac{dy}{dx} = 2sin2x(cos2x).(2) = 4sin2xcos2x$$

$$\frac{d^2y}{dx^2} = 4[sin2x(-2sin2x) + cos2x(2cos2x)] = -8\sin^2 2x\, cos2x + 8\cos^2 2x$$

EXAMPLE: If $y = lnx^2$, find d^2y/dx^2

Solution:

$$\frac{dy}{dx} = \frac{1}{x}.2x = \frac{2x}{x^2} = \frac{2}{x}$$

$$\frac{d^2y}{dx^2} = 2(-1.x^{-1-1}) = -2x^{-2} = \frac{-2}{x^2}$$