SAWA UNIVERSITY

FIRST ORDER DIFFERENTIAL EQUATIONS
المعادلات التفاضلية من الرتبة الأولى

Sawa University

Engineering Technology

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Lecture 2

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Lecture 2

First order differential equations المعادلات التفاضلية من الرتبة الأولى

يمكن كتابة معادلات هذا النوع من المعادلات التفاضلية بالأشكال الاتية:

$$\mathbf{M(x,y) + N(x,y)\frac{dy}{dx} = 0}$$
$$\mathbf{M(x,y)\,dx + N(x,y)\,dy = 0}$$
$$\mathbf{\frac{dy}{dx} = f(x,y)}$$

Solution methods differential equations of 1-st order

(1) Separable (Variable) equations المعادلات ذات المتغيرات القابلة للانفصال

(2) Homogeneous equations المعادلات المتجانسة

(3) Exact equation المعادلات التامة

(4) Linear equation المعادلات الخطية

1- Separable (Variable) equations

The general form of this method is:

$$\boxed{\mathbf{M(x)\,dx + N(y)\,dy = 0}}$$

Where M(x) is a function of (x) and N(y) is a function of (y) . in this method , the solution is by integrate M(x) with respect to variable (x) and N(y) with respect to variable (y) , the result is equation that relates y with x .

تكون صيغة المعادلة في هذه الطريقة $M(x)\,dx + N(y)\,dy = 0$ حيث $M(x)$هي دالة لـ (x) و $N(y)$ هي دالة لـ(y) والحل في هذه الطريقة نقوم اولاً بوضع كل متغير في طرف ونأخذ التكامل للطرفين حيث نكامل الدالة $M(x)$ بالنسبة لـ (x)والدالة $N(y)$ بالنسبة لـ (y) كما في الامثلة الاتية:

Ex1: solve the differential equation $\frac{dy}{dx} = 1 + x$

Sol:

$$\frac{dy}{dx} = 1 + x$$

$$dy = (1 + x)dx$$

$$\int dy = \int (1 + x)dx$$

$$y = x + \frac{x^2}{2} + c$$

Ex2: solve the differential equation $\frac{x}{y} = \frac{dy}{dx}$

Sol:

$$xdx = ydy$$

$$\int x\, dx = \int y\, dy$$

$$\frac{x^2}{2} + c = \frac{y^2}{2}$$

$$y^2 = x^2 + C'$$

$$y = \pm\sqrt{x^2 + C'}$$

2- Homogeneous First Order Differential Equation

The differential equation is called homogeneous and can be solved in this way if it satisfies the following condition:

$f(\gamma x, \gamma y) = f(x, y)$ This type is solved as follow:

$$V = \frac{y}{x} \rightarrow y = Vx \rightarrow \frac{dy}{dx} = V + x\frac{dv}{dx}$$

تكون المعادلة التفاضلية متجانسة إذا كانت الدالة $M(x, y)$ متجانسة والدالة $N(x, y)$ متجانسة أيضا وبنفس الدرجة.

ويتم التحقق من ذلك من خلال تعويض عن كل x ب (λx) وعن كل y ب (λy) فتحقق ما يلي:

$\boldsymbol{M(x, y)dx = N(x, y)dy}$ *is homogenous equation if*:

$M(\lambda x, \lambda y) = \lambda^n . M(x, y)$ the *part is homogenous*

$N(\lambda x, \lambda y) = \lambda^n . N(x, y)$ the *part is homogenous*

Ex 3: Prove that the function is homogenous:

$$\frac{dy}{dx} = \frac{x - y}{x + y}$$

Sol:

$(x + y)\, dy = (x - y)dx$

$M(x, y) = (x - y)$

$M(\lambda x, \lambda y) = (\lambda x - \lambda y)$

$= \lambda(x - y) = \lambda\, M(x, y) \quad \therefore hom.$

$N(x, y) = (x + y)$

$N(\lambda x, \lambda y) = (\lambda x + \lambda y) = \lambda(x + y) = \lambda N(x, y) \quad \therefore hom.$

Ex 4: solve the following

$xdy = (y - x)dx$

Sol: $\frac{dy}{dx} = \frac{y\gamma - \gamma x}{x\gamma} = \frac{y-x}{x}$ then this equation is homogeneous

Let $V = \frac{y}{x} \rightarrow y = Vx \rightarrow \frac{dy}{dx} = V + x\frac{dV}{dx}$

$V + x\frac{dV}{dx} = \frac{Vx-x}{x} \rightarrow V + x\frac{dV}{dx} = V - 1$

$x\frac{dV}{dx} = -1 \rightarrow dv = \frac{-dx}{x}$

$\int dv = \int \frac{-dx}{x}$

$v = -\ln(x) + c$

$But\ v = \frac{y}{x}\ then$

$\frac{y}{x} = -\ln(x) + c$

$y = -x\ln(x) + cx$

3-Exact First Order Differential Equation:

If An expression $M(x,y)dx + N(x,y)dy$ is an exact differential in a region R corresponding to the differential of some function $f(x,y)$. A first-order DE of the form $M(x,y)\,dx + N(x,y)\,dy = 0$ is said to be an exact equation, if the left side is an exact differential.

The differential equation that can be solved by this way must be satisfied the following condition:

For the D.E. $M(x,y)dx + N(x,y)dy = 0$ the exact solution can be applied if $\frac{dM}{dy} = \frac{dN}{dx}$ and the solution of differential equation is

بالنسبة للمعادلة التفاضلي بالنسبة بالنسبة للمعادلة التفاضلية، يمكن تطبيق الحل التام إذا تحقق الشرط $\frac{dM}{dy} = \frac{dN}{dx}$ ويكون حل المعادلة التفاضلية هو

$$F = \int Mdx + \int ndy$$

Or $F = \int Ndy + \int mdx$

Where (n) represents all polynomials of (N) that don't have the variable (x), and (m) represents all polynomials of (M) that don't have the variable (y).

1- Integrate (M(x,y),dx) with respect to (x).
2- Differentiate the result from step (1) with respect to (y).
3- Set the expression from step (2) equal to N(x,y).

1- نكامل $M(x,y)\,dx$ بالنسبة لـx
2- نشتق المعادلة في النقطة الأولى بالنسبة لـy $\frac{d}{dy}$
3 - نساوي المعادلة في النقطة 2 مع ال$N(x,y)$

Ex5: Find the solution of the equation:

$$3x(xy - 2)\,dx + (x^3 + 2y)\,dy = 0$$

Sol:

$M(x,y) = 3x(xy - 2)$

$M(x,y) = 3x^2y - 6x$

$\frac{dM}{dy} = 3x^2$

$N(x,y) = x^3 + 2y$

$\frac{dN}{dx} = 3x^2$

the D.E. is exact and can solved as follow:

$M(x,y) = \int (3x^2y - 6x)\,dx$

$M(x,y) = 3y \cdot \frac{x^3}{3} - 6 \cdot \frac{x^2}{2}$

$= y x^3 - 3x^2 + g(y)$

$\frac{dM}{dy} = x^3 + g'(y)$

But from $N(x,y)$:

$x^3 + g'(y) = x^3 + 2y \quad \Rightarrow g'(y) = 2y$

$g(y) = y^2$

$F(x,y) = yx^3 - 3x^2 + y^2 = c$

Ex6: Find the solution of the equation:

$$X\frac{dy}{dx} = -y - 4$$

$$xdy + (y+4)dx = 0$$

$$(y+4)dx + \; xdy \; = 0$$

$$M(x,y) = (y+4) \quad , \qquad \frac{dM}{dy} = 1$$

$$N(x,y) = x \qquad , \qquad \frac{dN}{dX} = 1$$

the D.E. is exact and can solved as follow:

$$M(x,y) = \int (y+4)\, dx$$

$$F(x,y) = (yx + 4x + g(y)$$

$$\frac{dF}{dy} = \; x + g'(y)$$

But from $N(x,y)$:

$x + g'(y) = x \quad \Rightarrow g'(y) = x - x = 0$

$g(y) = c1 \quad$ *sub in eq.*

$F(x,y) = xy + 4x + c1 = c$

$xy + 4x + = c2 \qquad\qquad c2 = c - c1$

Home Work:

Solve (x+y)dx+(x+2y)dy=0

4-Linear First Order Differential Equation

Linear first-order ODEs given by the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Working Rule:

Step 1. Convert the given equation to the standard form $\frac{dy}{dx} + Py = Q$

Step 2. Find the integrating factor, I.F.= $\mu = \exp \int P(x)\, dx$

Step 3. Then the solution is $y\,(I.F.) = \int Q(x)(I.F.)dx + C$

Ex6: Find the general solution for

$$\frac{dy}{dx} + 3ycosx = cosx$$

Sol:

$$\rho = e^{\int 3cosxdx} = e^{3sinx}$$

$$y = \frac{1}{e^{3sinx}} \int e^{3sinx} cosx\, dx \rightarrow y = \frac{1}{e^{3sinx}} \left[\frac{1}{3} e^{3sinx} + c\right] = \frac{1}{3} + ce^{-3sinx}$$

Ex7: Find the general solution for

$$\frac{dy}{dx}+\frac{1}{x}y=x^2-3 \quad ?$$

$$I.F.=\mu=\ \exp\int\frac{1}{x}dx=\ \exp\ \ln x=x$$

$$y\,\mathrm{x}=\int\ (x^2-3)\ xdx+C$$

$$\therefore\ yx=\frac{x^4}{4}-\frac{3x^2}{2}+C.$$

Home Work:

Solve the D.E. $\frac{1}{x}\mathbf{y'+4y=2}$

Thank You