SAWA UNIVERSITY

HIGHER-ORDER NON-HOMOGENEOUS LINEAR DIF.

Sawa University

Engineering Technology

Department of Refrigeration and Air-Conditioning Engineering

$3^{rd} Year$

Lecture 6

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2-Higher-Order Non-Homogeneous Linear Des

In Higher-Order Homogeneous Linear, we give methods for obtaining a particular solution yp once yc is known

كما في المعادلات الخطية المتجانسة ذات الرتبة العليا Higher-Order Homogeneous Linear equation

EX1:

$y''' + y'' - 2y' = \cos x$

1 − Find Homogeneous solution

$r^3 + r^2 - 2r = 0$

$$r(r^2 + r - 2) = r(r+2)(r-1) = 0$$

r 1 = 0

r 2 = -2

r 3 = 1

$$\boxed{y_h = C_1 + C_2 e^{-2x} + C_3 e^{x}}$$

2 − Find particular solution

$yp = A\cos x + B\sin x$

$$y_p' = -A\sin x + B\cos x$$

$$y_p'' = -A\cos x - B\sin x$$

$$y_p''' = A\sin x - B\cos x$$

$yp''' + yp'' - 2yp' = \cos x$

$(A\sin x - B\cos x) + (-A\sin x - B\cos x) - 2(-A\sin x + B\cos x) = \cos x$

$A\sin x - B\cos x - A\cos x - B\sin + 2A\sin x - 2B\cos x = \cos x$

$(-B - A - 2B)Cos\, x = 1\, Cos\, x$

$(-A - 3B) = 1 \ldots\ldots\ldots\ldots\ldots..(1)$

$(A - B + 2A)Sinx = 0$

$(3A - B) = 0$ ....(2)

From the second equation: $B = 3A$

Substitute into the first:

$-A-3(3A)=1 \Rightarrow -10A=1 \Rightarrow A=-\frac{1}{10}$

Then:

$$B = 3A = -\frac{3}{10}$$

General Solution:

$$\boxed{y = C_1 + C_2e^{-2x} + C_3e^{x} - \frac{1}{10}\cos x - \frac{3}{10}\sin x}$$

EX2:

$y^{(4)} - 3y'' + 2y = e^{2x}$

$1 -$ Find Homogeneous solution

$m^4 - 3m^2 + 2 = 0$

$(m^2 - 1)(m^2 - 2) = 0$

$m^2 - 1 = 0 \Rightarrow m^2 = 1 \quad \Rightarrow \qquad m = \pm 1$

or

$m^2 - 2 = 0 \Rightarrow m^2 = 2 \quad \Rightarrow \qquad m = \pm\sqrt{2}$

$m1 = 1$

$m\,2 = -1$

$m\,3 = +\sqrt{2}$

$m\,4 = -\sqrt{2}$

$$y_h = C_1e^x + C_2e^{-x} + C_3e^{\sqrt{2}x} + C_4e^{-\sqrt{2}x}$$

$2 -$ Find particular solution

$y_p = Ae^{2x}$

$y'_p = 2Ae^{2x}$

$y''_p = 4Ae^{2x}$

$y'''_p = 8Ae^{2x}$

$y^{(4)}{}_p = 16Ae^{2x}$

Substitute into the equation:

$$16Ae^{2x} - 3(4Ae^{2x}) + 2(Ae^{2x}) = e^{2x}$$

$$16A - 12A + 2A = 1 \implies 6A = 1 \implies A = \frac{1}{6}$$

$$y_p = \frac{1}{6}e^{2x}$$

General solution of the complete equation

$$\boxed{y = C_1e^{x} + C_2e^{-x} + C_3e^{\sqrt{2}x} + C_4e^{-\sqrt{2}x} + \frac{1}{6}e^{2x}}$$

Home work:

For each of the following problem find the general solution of the differential equation:

1- $y''' - 3y'' + 3y' - y = x^2 + 2x + 1$

2- $y^{(4)} - 2y'' + y = \cos(2x)$

Engineering Applications of High-Order Linear Differential Equations

High-order linear differential equations appear in engineering systems that contain **multiple energy storage elements**, such as:

1. Electrical Circuits

In a series or parallel RLC circuit, the sum of voltages equals the applied voltage E(t)(Kirchhoff's Voltage Law):

$$L\frac{d^2i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{1}{C}i(t) = E(t)$$

i(t) :Electric current (A)
L :Inductance (H)
R :Resistance (Ω)
C :Capacitance (F)
E(t) :Applied voltage (V)

2. Mechanical Vibrations

For a mass–spring–damper system, the external force F(t) equals the sum of forces from the mass, damper, and spring:

$$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + kx(t) = F(t)$$

x(t) :Displacement (m)

m : Mass (kg)

c : Damping coefficient (N·s/m)

k : Spring constant (N/m)

F(t) : Applied force (N)

Thank You