SAWA UNIVERSITY

FOURIER SERIES ENGINEERING APPLICATION
متسلسلة فورير

Sawa University

Engineering Technology

Department of Refrigeration and Air-Conditioning Engineering

$3^{rd} Year$

Lecture 8

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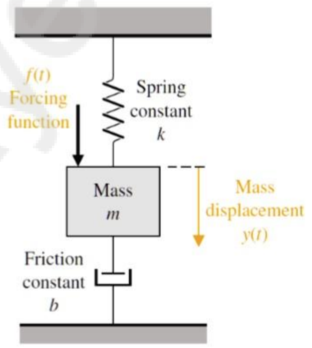
Lecture 8

Fourier Series متسلسلة فورير

Engineering Application

Application in Mechanical Vibration (Mass -Spring system)

$My'' + Cy' + Ky = f(t)$ المعادلة العامة



$y(t) = y_h + y_p$

$Y_{(t)} = S_0 * M^* \, Sin(wt-\propto)$ دالة فردية

$Y_{(t)} = S_0 * M^* \, Cos(wt-\propto)$ دالة زوجية

$$S_0 = \frac{F_0}{k}$$

S_0 تمثل الإزاحة الأساسية الناتجة عن القوة إذا لم يكن هناك إخماد

F_0 هي القيمة المضروبة بالدالة المثلثية في معادلة فوريير

K ثابت النابض

$$M^* = \frac{1}{\sqrt{(1-\frac{W^2}{W_0^2})^2 + (4\frac{C^2}{C_c^2} * \frac{W^2}{W_0^2})}} = Maginfiction\ factor$$

في هذه المسائل سوف نجد قيمة f(t) بأستخدام معادلة فوريير وسوف تنتج معادلة غير متجانسة من الدرجة الثانية ذات المعاملات الثابتة

$$\propto = tan^{-1}\frac{(c*w)}{k-W^2*M}$$

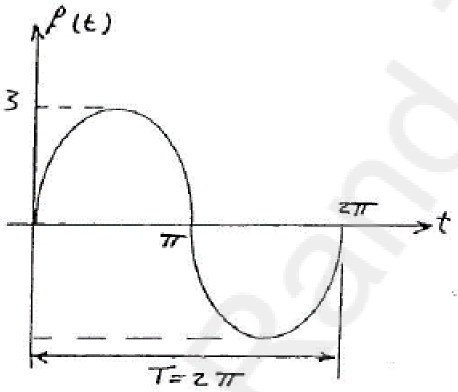
ملاحظة مهمة جدا :يجب وضع الحاسبة على نظام (rad) عند ايجاد ∝

$C_c = 2\sqrt{k.M}$ معامل الأخماد الحرج

W_0: frequency = $\sqrt{\frac{k}{m}}$

$W = 2n\pi$

ملاحظة :-

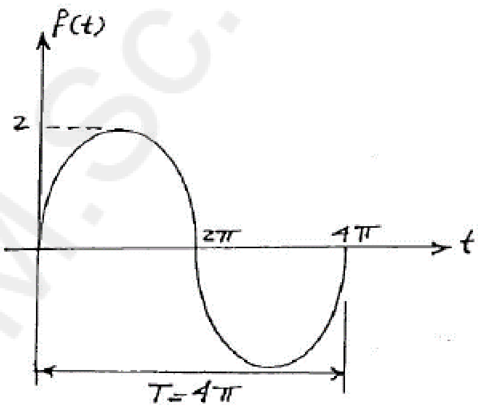


$F(t) = F_o \; Sin \; wt$

$\omega = \frac{2\pi}{T}$

$\omega = \frac{2\pi}{2\pi} = 1$

$\therefore F(t) = 3 \; Sin t$



$F(t) = F_o \; Sin \; wt$

$w = \frac{2\pi}{T} = \frac{2\pi}{4\pi} = \frac{1}{2}$

$\therefore F(t) = 2 \; Sin \frac{t}{2}$

Ex: A lightly damped spring-mass system is governed by the equation

$$My'' + Cy' + Ky = f(t)$$

- Mass: M=1 kg
- Damping coefficient: C=0.5 Ns/m
- Spring constant: K=4 N/m
- External force (periodic) expressed as a simple Fourier series:

$$f(t) = 2 + 3cos(t) + sin(2t)$$

Find the complete response y(t)of the system.

Sol:

Natural frequency and critical damping

$W_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{4}{1}} = 2$ rad/s

$$C_c = 2\sqrt{k.M} = 2\sqrt{4 * 1} = 2 * 2 = 4$$

Solve for each Fourier component separately

(a) Constant component

F_0=2

$$S_0 = y_0 = \frac{F_0}{k} = \frac{2}{4} = 0.5$$

(b) Cosine component

$F_1\cos(t)$, F_1=3 , ω_1=1

$$S_1 = y_1 = \frac{F_1}{k} = \frac{3}{4}$$

$$M^* = \frac{1}{\sqrt{(1-\frac{W^2}{W_0^2})^2 + (4\frac{C^2}{C_c^2} * \frac{W^2}{W_0^2})}}$$

$$M^* = \frac{1}{\sqrt{(1-\frac{1^2}{2^2})^2 + (4\frac{(0.5)^2}{(4)^2} * \frac{1^2}{2^2})}} = 1.28$$

$$\propto = tan^{-1}\frac{(c*w)}{k-W^2*M} = tan^{-1}\frac{(0.5*1)}{4-1^2*1} = 0.165$$

$$Y1_{(t)} = S_1 * M^* \, Cos(wt - \propto) = \frac{3}{4} * 1.28 \, Cos(1*t - 0.165)$$

$$=0.960 \, Cos(t - 0.165)$$

(b) sine component

$F_2\sin(t)$, $F_2=1$, $\omega_2=2$

$$S_2 = y_2 = \frac{F_2}{k} = \frac{1}{4}$$

$$M^* = \frac{1}{\sqrt{(1-\frac{W^2}{W_0^2})^2 + (4\frac{C^2}{C_c^2} * \frac{W^2}{W_0^2})}}$$

$$M^* = \frac{1}{\sqrt{(1-\frac{2^2}{2^2})^2 + (4\frac{(0.5)^2}{(4)^2} * \frac{2^2}{2^2})}} = 4$$

$$\propto = tan^{-1}\frac{(c*w)}{k-w^2*M} = tan^{-1}\frac{(0.5*2)}{4-2^2*1} = tan^{-1}(\infty) = \frac{\pi}{2}$$

$$Y2_{(t)} = S_2 * M^* \, Cos(wt - \propto) = \frac{1}{4} * 4 \, Sin\left(2 * t - \frac{\pi}{2}\right)$$

$$= Sin\left(2t - \frac{\pi}{2}\right)$$

$$y(t) = y0 + y1(t) + y2(t) \approx 0.5 + 0.855 \, Cos(t - 0.165) + 1 sin(2t - \pi/2)$$

Thank You