SAWA UNIVERSITY

LAPLACE TRANSFORMATION FOR SOLVING DIFFERENTIAL EQUATIONS AND ENGINEERING APPLICATIONS

Sawa University

Engineering Technology

Department of Refrigeration and Air-Conditioning Engineering

$3^{rd} Year$

Lecture 11

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Lecture 11

-Laplace Transformation for Solving Differential Equations and Engineering Applications

$$L\{y'(t)\} = sY(s) - y(0)$$

$$L\{y'(t)\} = s\,L\{y(t)\} - y(0)$$

$$L\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$$

$$L\{y''(t)\} = s^2L\{y(t)\} - sy(0) - y'(0)$$

$$L\{y'''(t)\} = s^3L\{y(t)\} - s^2y(0) - sy'(0) - y''(0)$$

$$L\{y^{(n)}(t)\} = s^nL\{y(t)\} - s^{n-1}y(0) - s^{n-2}y'(0) - y^{(n-1)}(0)$$

Example1:

$$y' + y = 2, \quad y(0) = 1$$

$$L\{y'\} + L\{y\} = L\{2\}$$

$$(sY(s) - y(0)) + Y(s) = \frac{2}{s}$$

$$(sY(s) - 1) + Y(s) = \frac{2}{s}$$

$$(s+1)Y(s) - 1 = \frac{2}{s}$$

$$(s+1)Y(s) = \frac{2}{s} + 1$$

$$Y(s) = \frac{2/s + 1}{s+1} = \frac{2}{s(s+1)} + \frac{1}{s+1}$$

$$\frac{2}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$2 = A(s+1) + Bs$$

$$2 = As + A + Bs$$

$$As + Bs = 0 \Rightarrow A + B = 0$$

$$A = 2 \ sub \ in \ eq \ (1)$$

$$A + B = 0$$

$$2 + B = 0 \Rightarrow B = -2$$

$$\frac{2}{s(s+1)} = \frac{2}{s} - \frac{2}{s+1}$$

$$Y(s) = \frac{2}{s} - \frac{2}{s+1} + \frac{1}{s+1} = \frac{2}{s} - \frac{1}{s+1}$$

$$y(t) = L^{-1}\left\{\frac{2}{s}\right\} - L^{-1}\left\{\frac{1}{s+1}\right\} = 2 - e^{-t}$$

$$\boxed{y(t) = 2 - e^{-t}}$$

Example 2:

$$y' + 3y = 6, \quad y(0) = 2$$

$$L\{y'\} + 3L\{y\} = L\{6\}$$

$$(sY(s) - y(0)) + 3Y(s) = \frac{6}{s}$$

$$(sY(s) - 2) + 3Y(s) = \frac{6}{s}$$

$$(s+3)Y(s) - 2 = \frac{6}{s} \quad \Rightarrow \quad (s+3)Y(s) = \frac{6}{s} + 2$$

$$Y(s) = \frac{6/s + 2}{s+3} = \frac{6}{s(s+3)} + \frac{2}{s+3}$$

$$\frac{6}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3}$$

$$6 = A(s+3) + Bs$$

$$6 = As + 3A + Bs$$

$$As + Bs = 0 \Rightarrow A + B = 0$$

$$3A = 6 \Rightarrow A = 2 \ sub\ in\ eq\ (1)$$

$$A + B = 0$$

$$2 + B = 0 \Rightarrow B = -2$$

$$\frac{6}{s(s+3)} = \frac{2}{s} - \frac{2}{s+3}$$

$$Y(s) = \frac{2}{s} - \frac{2}{s+3} + \frac{2}{s+3} = \frac{2}{s}$$

$$y(t) = L^{-1}\left\{\frac{2}{s}\right\} = 2$$

Example 3:

$$y'' + 3y' + 2y = 6 \qquad y(0) = 2, \qquad y'(0) = 0$$

$$s^2Y(s) - sy(0) - y'(0) + 3[sY(s) - y(0)] + 2Y(s) = \frac{6}{s}$$

Substitute initial values y(0)=2, y′(0)=0

$$s^2Y(s) - s\,y(0) - y'(0) + 3sY(s) - 3y(0) + 2Y(s) = \frac{6}{s}$$

$$(s^2 + 3s + 2)Y(s) - s\,y(0) - y'(0) - 3y(0) = \frac{6}{s}$$

$$(s^2 + 3s + 2)Y(s) = \frac{6}{s} + s\,y(0) + y'(0) + 3y(0)$$

$$Y(s) = \frac{\frac{6}{s} + s\,y(0) + y'(0) + 3y(0)}{s^2 + 3s + 2}$$

$$Y(s) = \frac{\frac{6}{s} + 2s + 6}{(s+1)(s+2)}$$

$$\frac{2s^2 + 6s + 6}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}.$$

$$2s^2 + 6s + 6 = A(s+1)(s+2) + B\,s(s+2) + C\,s(s+1)$$

$$A(s+1)(s+2) = A(s^2+3s+2) = As^2+3As+2A.$$
$$Bs(s+2) = B(s^2+2s) = Bs^2+2Bs,$$
$$Cs(s+1) = C(s^2+s) = Cs^2+Cs.$$

$$2s^2+6s+6 = (A+B+C)s^2+(3A+2B+C)s+2A.$$

A + B + C = 2 …….(1)
3A + 2B + C = 6 …..(2)
2A = 6 ……..(3) ⇒A=3.

Substitute $A = 3$ into the first two equations:

$$\begin{cases} 3+B+C=2 \Rightarrow B+C=-1 \quad (1) \\ 3(3)+2B+C=6 \Rightarrow 9+2B+C=6 \Rightarrow 2B+C=-3 \quad (2) \end{cases}$$

From Eq. (1)

$B + C = -1 \quad \Rightarrow \quad B = -1 - C$ Sub in eq. (2)

$2(-1-C) + c = -3 \Rightarrow -2-2c+c = -3$

$-c = -3+2 \Rightarrow -c = -1 \Rightarrow c = 1$

Sub:

$B = -1 - C \quad \Rightarrow \quad B = -1-(1) = -2$

$$A = 3, \quad B = -2, \quad C = 1.$$

$$Y(s) = \frac{3}{s} - \frac{2}{s+1} + \frac{1}{s+2}.$$

$$L^{-1}\left\{\frac{1}{s}\right\}=1, \quad L^{-1}\left\{\frac{1}{s+a}\right\}=e^{-at}.$$

$$y(t)=3(1)-2e^{-t}+e^{-2t}.$$

$$\boxed{y(t)=3-2e^{-t}+e^{-2t}}$$

Thank You