SAWA UNIVERSITY

RUNGE- KUTTA METHOD

Sawa University

Engineering Technology

Department of Refrigeration and Air-Conditioning Engineering

$3^{rd} Year$

Lecture 16

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Lecture 16

3.3 Runge-Kutta Method

تعتبر هذه الطريقة أكثر دقة من الطرق السابقة وتوجد لها عدة صيغ من أشهرها الصيغة ذات المرتبة الرابعة والتي يكون القانون العام :

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where:

$$k_1 = h \cdot f(x_i, y_i)$$

$$k_2 = h.f\left(xi + \frac{h}{2}, yi + \frac{k_1}{2}\right)$$

$$k_3 = h.f\left(xi + \frac{h}{2}, yi + \frac{k_2}{2}\right)$$

$$k_4 = h.f\left(xi + h, yi + k_3\right)$$

$$f(x,y) = \dot{y}$$

Example (1): Find the solution of differential equation by using Runge-Kutta Method at $x = 0.1$, $\frac{dy}{dx} = \dot{y} = x + y$, use $y(0) = 1$, $h = 0.1$.

Solve:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$For\ i = 0:\ x_o = 0,\ y_o = 1,\ h = 0.1$

$$y_1 = y_o + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$f(x,y) = \dot{y} = x + y$$

$$k_1 = h \cdot f(x_o, y_o) = h \cdot f(0, 1) = 0.1 \times (0 + 1) = 0.1$$

i	x_i	y_i $(Num.)$
0	0	1
1	0.1	1.11034

$$k_2 = h.f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = k_2 = h.f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}\right)$$

$=0.1*(0.05+1.05)=0.11$

$$k_3 = h.f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = k_3 = h.f\left(0 + \frac{0.1}{2}, 1 + \frac{0.11}{2}\right)$$

$=0.1*(0.05+1.055)=0.1105$

$$k_4 = h.f\left(x_0 + h, y_0 + k_3\right) = k_4 = h.f\left(0 + 0.1, 1 + 0.1105\right)$$

$=0.1*(0.1+1.155)=0.12105$

$$y_1 = y_1 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$y_1 = 1 + \frac{1}{6}\left(0.1 + 2*0.11 + 2*0.1105 + 0.12105\right)$$

$for\, i = 1 : x_1 = x_0 + h = 0.1,\ y_1 = 1.11034$

Example (2):

Find the solution of differential equation by using **Runge-Kutta Method at $x = 0.1$**

$$\frac{dy}{dx} = y' = x + 1, \qquad \text{use } y(0) = 1, \qquad h = 0.1$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

1. $k_1 = hf(x_0, y_0) = 0.1 \cdot (0 + 1) = 0.1$
2. $k_2 = 0.1 \cdot f(0 + 0.05, 1 + 0.1/2) = 0.1 \cdot f(0.05, 1.05) = 0.1 \cdot (0.05 + 1) = 0.105$
3. $k_3 = 0.1 \cdot f(0.05, 1 + 0.105/2) = 0.1 \cdot f(0.05, 1.0525) = 0.1 \cdot (0.05 + 1) = 0.105$
4. $k_4 = 0.1 \cdot f(0.1, 1 + 0.105) = 0.1 \cdot f(0.1, 1.105) = 0.1 \cdot (0.1 + 1) = 0.11$

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = 1 + \frac{1}{6}(0.1 + 2(0.105) + 2(0.105) + 0.11)$$

$$y_1 = 1 + \frac{1}{6}(0.1 + 0.21 + 0.21 + 0.11)$$

$$y_1 = 1 + \frac{1}{6}(0.63)$$

$$y_1 = 1 + 0.105 = 1.105$$

Thank You